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Defect conformal field theory and locally localized gravity

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Abstract: Gravity may be "locally localized" over a wide range of length scales on a d-dimensional anti-de Sitter (AdS) brane living inside AdS_{d+1}. In this paper we examine this phenomenon from the point of view of the holographic dual "defect conformal field theory". The mode expansion of bulk fields on the gravity side is shown to be precisely dual to the "boundary operator product expansion" of operators as they approach the defect. From the field theory point of view, the condition for localization is that a "reduced operator" appearing in this expansion acquires negative anomalous dimension. In particular, a very light localized graviton exists when a mode arising from the reduction of the ambient stress-energy tensor to the defect has conformal dimension $\Delta \sim d-1$. The part of the stress tensor containing the defect dynamics has dimension $\Delta = d-1$ in the free theory, but we argue that it acquires a positive anomalous dimension in the interacting theory, and does not participate in localization in the regime of small backreaction of the brane. We demonstrate that such an anomalous dimension is consistent with the conservation of the full stress-energy tensor. Finally, we analyze how to compute the anomalous dimensions of reduced operators from gravity at leading order in the interactions with the brane.

Keywords: Brane Dynamics in Gauge Theories, AdS-CFT and dS-CFT Correspondence, Classical Theories of Gravity.

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1. Introduction and summary

It was suggested a long time ago [1] that the 3+1-dimensional world that we perceive around us could be merely a subspace of a higher-dimensional non-compact universe, provided the phenomena we observe are somehow localized to this subspace. The main obstacle to realizing this suggestion lies in reconciling it with the nature of gravity, which on the one hand is seen to display four-dimensional behavior over the entire range of known experiments, but on the other hand must propagate in all spacetime dimensions due to its geometrical character.

In recent years it has been realized that this obstacle could be overcome if the higher dimensional space is (asymptotically) anti-de Sitter (AdS) space. Randall and Sundrum demonstrated [2] that an effective lower-dimensional theory of gravity could be localized on a flat brane in anti-de Sitter space, providing an alternative to compactification. This solution involves a fine tuning between the brane tension and the bulk cosmological constant, so it was natural to ask what happens to localized gravity as the tension is detuned [3]–[6]. A larger tension leads to a de Sitter brane and more strongly localized gravity, but decreasing the tension produces an anti-de Sitter brane and the localized graviton apparently vanishes from the spectrum. However, it was shown in [7] that for a slight detuning, a massive mode of the graviton with a hierarchically small mass approximates lower-dimensional gravity over a range of length scales; this phenomenon was dubbed "locally localized gravity". So far a "local localization" of gravity by this phenomenon has only been observed in the context of low-energy effective field theory. It is interesting to study whether this phenomenon can also occur within a consistent theory of quantum gravity, such as string theory.

The AdS/CFT correspondence [8, 9] relates gravitational phenomena in $(d+1)$-dimensional anti-de Sitter space to dynamics in a non-gravitational d-dimensional conformal field theory, which can be visualized as living on the boundary of the anti-de Sitter space. For the flat brane (as in [2]) and de Sitter brane cases, the solution for the d-dimensional brane in AdS_{d+1} excises the boundary from the spacetime, so it is subtle to analyze these solutions in the context of the AdS/CFT correspondence. On the other hand, the solution for an anti-de Sitter brane preserves the boundary of the AdS space-time, intersecting it on a codimension one subspace.[1] Consequently, it was natural to conjecture that this geometry has a holographic description consisting of a field theory containing a defect or hypersurface, also codimension one, on which additional dynamics are localized. Because the anti-de Sitter isometries of the spacetime are broken to the smaller anti-de Sitter group of the brane, the dual field theory preserves scale invariance as part of a reduced conformal group.

An explicit realization of this correspondence in string theory involving a D3/D5-brane system in type-IIB string theory was suggested in [10]. In this set-up a stack of M D5-branes wraps $AdS_4 \times S^2$ inside the geometry $AdS_5 \times S^5$ with N units of five-form flux. This is the near-horizon limit of N D3-branes intersecting orthogonally with M D5-branes along a $2+1$ dimensional subspace. This system was studied in detail in [11], where the lagrangian for the dual "defect conformal field theory" (dCFT) was constructed, and conformal invariance was proven for the theory with U(1) gauge group. The superconformal symmetry for arbitrary gauge group was proven by [12]. Other studies of AdS/dCFT and defect quantum field theories (dQFTs) include [13]–[22]; there are many additional papers on the special case of AdS_2 branes in AdS_3.

One may then study whether the phenomenon of locally localized gravity is realized in the D3/D5 system. The gravitational approximation to its dynamics (far away from the D5-branes) is valid when $N \gg 1$ and $g_s N \gg 1$, where g_s is the string coupling. Local localization of gravity arises only when the backreaction of the branes, proportional to M, is taken into account. However, thus far the study of AdS/dCFT has only been possible in the limit of vanishing backreaction, $M \ll N$, where the gravity background may be treated as a set of probe D5-branes in a fixed anti-de Sitter space. The complete solution including the backreaction is unknown, although steps toward understanding it have been taken in [23]. This is an apparent obstacle to the search for local localization in string theory. The dual dCFT [11], on the other hand, is defined for all M and N, so one could hope to use the duality to study local localization without finding the backreacted gravity

[1] Note that the AdS brane is not itself a boundary of the space-time.

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solution. As a first step, one would like to address the question: what is the realization of local localization in a dual field theory? In this paper we will propose an answer to this question and justify our proposal.

The essential point is the familiar correspondence between the mass of a bulk mode and the conformal dimension of the dual operator. The $(d+1)$-dimensional graviton can be expanded in modes propagating on the d-dimensional AdS subspace, and local localization requires that a mode should be present that has almost vanishing AdS_d mass, hence approximating the behavior of a d-dimensional graviton. This mode implies in the dual field theory the existence of a $(d-1)$-dimensional spin-2 operator, resulting from some kind of decomposition of the d-dimensional stress tensor, with conformal dimension (under the reduced conformal group) approaching $\Delta = d-1$.

From the field theory point of view there seem to be two natural candidates for what this spin-2 operator, corresponding to the "locally localized stress tensor", should be. Generally the full stress tensor of a dQFT can be written in the form

$$T_{\mu\nu}(\vec{y},x) = \mathcal{T}_{\mu\nu}(\vec{y},x) + \delta(x)\,\delta^{\mu}_{k}\delta^{\nu}_{l}\,t_{kl}(\vec{y})\,, \tag{1.1}$$

where $\mathcal{T}_{\mu\nu}$ is the stress tensor in the absence of the defect, composed of d-dimensional fields only, while t_{kl} is the contribution from defect interactions, including both d- and $(d-1)$-dimensional fields. (Here and below $\mu,\nu,\ldots$ run over the d coordinates of the ambient theory while $k,l,\ldots$ run over the $(d-1)$ coordinates of the defect; we will label the defect coordinates by $\vec{y}$ and the coordinate perpendicular to the defect by x, with the defect at $x=0$.) One candidate for the locally localized stress tensor is a $(d-1)$-dimensional operator arising in a power-series expansion of the stress tensor $\mathcal{T}_{\mu\nu}$ of the d-dimensional fields around the defect. Another natural candidate is simply t_{kl}. We shall explore both possibilities.[2]

The possibility of the localized stress tensor arising in an expansion of $\mathcal{T}_{\mu\nu}$ is reminiscent of the localized graviton appearing in the mode expansion of the full graviton in the original model of [7]. We will show that the two are indeed related. There is a natural expansion for d-dimensional operators as they approach the defect, namely the so-called boundary operator product expansion (BOPE) that is used in quantum theories with planar boundary.[3] In the absence of interactions with the defect the BOPE is just the Taylor expansion of an operator in the coordinate perpendicular to the defect, and the expansion is just composed of the "reduced operators" $[\partial_x^n \mathcal{O}](\vec{y}) \equiv \partial_x^n \mathcal{O}(\vec{y},x)|_{x=0}$. The interacting BOPE generalizes the Taylor series and introduces both "reductions" of other d-dimensional operators and $(d-1)$-dimensional defect operators into the expansion. We demonstrate that the part of the BOPE involving reduced operators matches precisely with

[2] In the formula (1.1) we assumed that we have interactions localized on the defect; one could also study theories where the defects just give "boundary conditions" for the d-dimensional fields, as in the D3/NS5 system [24], and then the second term in (1.1) will not appear and we will only have the first possibility. We also assumed that we have the same ambient theory on both sides of the defect, though this does not have to be the case. All of our analysis in this paper will be valid in these other cases as well, with obvious small modifications.

[3] The term "defect operator product expansion" (dOPE) was considered and prudently rejected.

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the decomposition of AdS_{d+1} bulk fields into AdS_d modes in the gravity theory. The locally localized graviton of the model of [7] can hence be identified with an operator in the BOPE of the dual stress tensor; it is reasonable to expect that the generic dual to a theory of "locally localized gravity" will evince similar behavior. For weak coupling with the defect, the lowest dimension operator appearing in this BOPE has a dimension close to d, while a very light graviton requires a dimension close to $(d-1)$. Thus, strong coupling with the defect is required to realize locally localized gravity in this way, mirroring the requirement of large backreaction in the gravitational theory.

We also consider the operator t_{kl}. When the field theory is free, this operator has precisely the right dimension to correspond to an *exactly* massless localized graviton, so at first sight it seems a more natural source for localized gravity. The string theory translation of t_{kl} is different from a "locally localized graviton", however — instead, it is a spin-2 field living on the brane itself. Such a field is "naturally localized" on the brane, in contrast with the "locally localized" examples discussed above. Even though this possibility was not considered in [1], it has a similar phenomenology to the locally localized case, and it seems natural given our modern perspective of the many fields living on branes, such as open string modes on D-branes.

Naively, the conservation of the total stress tensor in (1.1) means that it has dimension d, and suggests that t_{kl} should have dimension $(d-1)$, meaning it should always correspond to a localized massless graviton. However, we find that despite the conservation of the total stress tensor, t_{kl} alone is not conserved and accordingly can acquire an anomalous dimension. We demonstrate this explicitly in a toy defect quantum field theory, as well as showing how the related divergence is canceled in correlators of the total stress tensor, as it must be since the total stress tensor is conserved.

In the case of the D3/D5 system, in the regime where gravity is a good description but the branes have small backreaction, there is no light spin-2 field living on the D5-branes. Thus, we argue that in this system t_{kl} acquires a very large anomalous dimension at large g_sN and small g_sM, and hence does not lead to a light localized graviton. This will always be the case in string theory in the limit of small backreaction, since none of the branes of string theory carry any light spin-2 fields in their worldvolume. Of course, to achieve localization we necessarily need to have large backreaction of the brane. In this regime it is possible that the field corresponding to t_{kl} could become light and become relevant for localized gravity. In general this field mixes with the operators coming from the expansion of $\mathcal{T}_{\mu\nu}$ near the brane, so the localized graviton could also come from a mixture of the two types of fields.

The next three sections of this paper will be devoted to justifying the statements made above, and computing in field theory the anomalous dimension of operators analogous to t_{kl}. In the last section we consider calculations in the supergravity limit that are useful for computing the anomalous dimensions of reduced operators at large g_sN when the backreaction is small. These involve the evaluation of two-point functions for d-dimensional operators in the presence of the brane. At leading order in the interactions with the brane, the Feynman diagrams with sources at the boundary of AdS space ("Witten diagrams") [25] generically have two internal integrations to be performed, and consequently they are

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similar to four-point functions in ordinary AdS/CFT. We employ the techniques of "without really trying" [26], developed for calculating four-point amplitudes, to evaluate the three types of diagrams contributing to two-point functions at this leading order, for the case of scalar fields. All diagrams are seen to produce contributions to the correlation functions with the form dictated by the reduced conformal invariance. In future work we hope to use these results to study whether localization of gravity occurs in the D3/D5 system.

Anomalous dimensions appear in perturbation theory multiplying logarithms of the coordinates in correlation functions. One can extract from four-point functions in a non-defect CFT the anomalous dimensions of operators by analyzing the limit as the operators approach each other in pairs using the ordinary OPE. Similarly, one may extract the anomalous dimensions in our case by examining a two-point function of d-dimensional operators in the limit as the two operators approach the defect at fixed distance from each other, using the BOPE. We find that generically reduced operators obtain anomalous dimensions, which are not related to the anomalous dimensions of the d-dimensional operators that they reduce from. This is due to the fact that, in the presence of the defect, the process of bringing an operator near the defect leads to divergences and a regularization is required, as for composite operators.

The organization of this paper is the following. In section 2 we review and elaborate on some basic facts about dCFTs, the BOPE, and conserved currents. In section 3 we demonstrate that the decomposition of bulk AdS_{d+1} fields into AdS_d modes directly implies the BOPE via the AdS/CFT correspondence. This leads us to our proposal for the field theory interpretation of local localization. In section 4 we perform field theory calculations in a toy dQFT to demonstrate how the delta-function supported part of a current can acquire an anomalous dimension without violating conservation for the total current. Finally, in section 5 we turn to the gravity calculations necessary to extract (in the appropriate limit) the anomalous dimensions of reduced operators. We evaluate the three possible classes of two-point functions and interpret the logarithms that appear in terms of anomalous dimensions. An appendix summarizes some conventions which are useful for section 4.

2. Ambient operators and defect operators in dCFT

2.1 Generalities

A d-dimensional conformal field theory has the spacetime symmetry group $\mathrm{SO}(d,2)$, which is also the isometry group of the AdS_{d+1} background of its gravity dual. Introducing a brane wrapped on an AdS_d submanifold, as in the system of [7], breaks the isometry group to $\mathrm{SO}(d-1,2)$. The AdS_d brane intersects the boundary in a codimension one hypersurface, inducing a spatial defect in the field theory dual. The symmetry group of the field theory is thus broken to the $(d-1)$-dimensional conformal group; although certain translations, rotations and special conformal transformations are lost, scale invariance is preserved. The dual is hence a defect conformal field theory (dCFT). Of course, even though the study of dCFTs is motivated by the fact that these theories naturally appear in the context of the AdS/CFT correspondence, one can also in principle construct such field theories directly.

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It is important to note that the preserved symmetry group does not only act on the directions along the defect; the scale transformation, for example, rescales the perpendicular direction as well as the defect directions, and the unbroken special conformal transformations also involve the perpendicular direction. The unbroken symmetries constitute the subgroup of d-dimensional conformal transformations preserving the defect $x = 0$, which is the smallest group containing the ISO$(d-2,1)$ Poincaré algebra as well as the d- (not $(d-1)$-) dimensional inversion transformation $\{x \to x/(x^2+\vec{y}^2); \vec{y} \to \vec{y}/(x^2+\vec{y}^2)\}$. Note that the inversion must be about a point inside the defect.

In the case of supersymmetric theories, the brane/defect generically breaks fermionic and R-symmetries as well. Our canonical example will be the defect superconformal theory descending from $\mathcal{N}=4$ super Yang-Mills in four dimensions, which is dual to type-IIB string theory on $AdS_5 \times S^5$ with N units of five-form flux and M D5-branes wrapped on $AdS_4 \times S^2$ [7, 11, 24]. In this case the SU$(2,2|4)$ supergroup is broken to OSp$(4|4)$; besides the reduction of the conformal/AdS group from SO$(4,2)$ to SO$(3,2)$, the R-symmetry is broken from SU(4) to SU$(2)_V \times$ SU$(2)_H$, and the number of both ordinary and conformal supercharges is cut in half.

When a dCFT has a lagrangian description, as is the case for this example, the action can be written explicitly in terms of "ambient" fields $\phi_d(\vec{y},x)$ propagating throughout space as well as "defect" fields $\chi_{d-1}(\vec{y})$ confined to the defect. The ambient and defect fields are coupled on the defect:

$$\mathcal{L} = \mathcal{L}_{amb}\left[\phi_d(\vec{y},x)\right] + \delta(x)\,\mathcal{L}_{def}\left[\phi_d(\vec{y},0), \chi_{d-1}(\vec{y})\right], \tag{2.1}$$

where $\mathcal{L}_{amb}$ is the lagrangian of the parent theory in the absence of the defect, and $\mathcal{L}_{def}$ contains couplings that explicitly break the symmetries down to the preserved subgroup.[4]

The defect fields transform under the preserved symmetry group as in an ordinary conformal field theory. From them we may form gauge-invariant operators $\mathcal{O}_{d-1}(\vec{y})$ which will form representations of the $(d-1)$-dimensional conformal algebra; these operators could also contain the restrictions $\phi_d(\vec{y},0)$ of ambient fields to the defect. We can also assemble gauge-invariant operators $\mathcal{O}_d(\vec{y},x)$ from ambient fields only, which are defined throughout space. The restrictions of these ambient operators to the defect produces "reduced operators" $[\mathcal{O}_d](\vec{y}) \equiv \mathcal{O}_d(\vec{y},x)|_{x\to 0}$. In the free theory the two types of $(d-1)$-dimensional operators, $\mathcal{O}_{d-1}(\vec{y})$ and $[\mathcal{O}_d](\vec{y})$, are clearly distinguishable, but once we add interactions they can mix together, and we will call all of them "defect operators" (since they are parameterized by a position along the defect). The correlation functions of these operators among themselves obey the usual constraints of $(d-1)$-dimensional conformal symmetry; in particular, the two-point and three-point functions of primary operators are determined (up to a constant) in terms of the scaling dimensions of the operators, as in an ordinary CFT.

The constraints placed on correlation functions involving the ambient operators $\mathcal{O}_d(\vec{y},x)$ by the defect conformal group were analyzed in [27] in the context of a conformal field theory with a boundary, and the same analysis applies to our case. The reduced con-

[4]There are also examples of dCFTs with codimension bigger than 1, see for instance [21]. We will only discuss the codimension 1 case here, though the generalization of our results to arbitrary codimension should be straightforward.

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formal algebra allows a non-zero one-point function for primary scalar ambient operators $\mathcal{O}_d$ of scaling dimension Δ, of the form

$$\langle \mathcal{O}_d(\vec{y}, x) \rangle = \frac{A_{\mathcal{O}_d}}{x^{\Delta}} , \tag{2.2}$$

for some constant[5] $A_{\mathcal{O}_d}$. Mixed two-point functions of ambient operators of dimension Δ and defect operators of dimension Δ' are also uniquely determined up to a constant, taking the form

$$\left\langle \mathcal{O}_d(\vec{y}, x)\, \mathcal{O}_{d-1}(\vec{y}') \right\rangle = \frac{B_{\mathcal{O}_d \mathcal{O}_{d-1}}}{x^{\Delta-\Delta'} (x^2 + (\vec{y} - \vec{y}')^2)^{\Delta'}} , \tag{2.3}$$

for some constant $B_{\mathcal{O}_d \mathcal{O}_{d-1}}$. The simplest allowed correlators of ambient fields which have some freedom in their coordinate dependence are two-point functions of ambient scalar operators, which have the form

$$\left\langle \mathcal{O}^1_d(\vec{y}, x)\, \mathcal{O}^2_d(\vec{y}', x') \right\rangle = \frac{1}{x^{\Delta_1} (x')^{\Delta_2}} f(\xi) , \tag{2.4}$$

where $f(\xi)$ is an arbitrary function of the conformally invariant variable

$$\xi \equiv \frac{(\vec{y} - \vec{y}')^2 + (x - x')^2}{4xx'} . \tag{2.5}$$

2.2 Boundary operator product expansion

Before we inserted the defect, the ambient operators $\mathcal{O}_d(\vec{y}, x)$ transformed in representations of the full (super)conformal algebra. In the presence of the defect we can decompose these into representations of the defect (super)conformal algebra and hence it is natural to look for an appropriate way to write any ambient operator in terms of a set of defect operators.

Since translations in x are no longer a symmetry, we cannot use them to relate operators at different values of x. We can still use the scaling transformations to relate operators at different non-zero values of x, so these are all in the same representation of the conformal group. However, the reduced operators $[\mathcal{O}_d](\vec{y}) \equiv \mathcal{O}_d(\vec{y}, x)|_{x\to 0}$ are no longer related by any symmetry to the operators $\mathcal{O}_d(\vec{y}, x \neq 0)$. In particular, the reduced operators can mix with defect operators $\mathcal{O}_{d-1}(\vec{y})$. It seems that even though the operators $\mathcal{O}_d(\vec{y}, x \neq 0)$ can have well-defined scaling dimensions, they are never primaries of the defect conformal algebra, since they are not annihilated by the preserved special conformal generators.

[5] In general we could have a different constant appearing here for positive and negative values of x, and we should write this as $A_{\mathcal{O}_d}/|x|^{\Delta}$ since Δ is not necessarily integer. This is the form we would generally find from explicit computations like the ones we perform in the later sections. In the D3/D5 system which we will mainly be interested in, the chiral operators in the bulk all have integer dimensions, and there is a parity symmetry (discussed in [11]) which relates positive and negative values of x and which allows only even-dimensional operators to obtain one-point functions, so the expression (2.2) is always valid and the one-point function is always an analytic function of x. In this particular system it seems that also the two-point functions discussed below are always functions of x (as we will write them) and not of $|x|$. However, this need not be the case in general (and it cannot be the case when non-integer dimensions are involved). We will generally ignore this subtlety in this paper.

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Thus, if we want to discuss the $\mathcal{O}_d$ using the usual representations of the unbroken conformal algebra which are built from (quasi-)primary operators, we need to do something else.

In order to discuss the behavior of ambient operators near the defect, it is convenient to introduce the notion of an expansion in a series of defect operators, which we will call the BOPE since it has similar properties to the boundary operator product expansion discussed in [27] (and in many other places for two dimensional conformal field theories). By the usual arguments of local field theories, we can expand ambient operators as $x \to 0$ as a power series in operators $\mathcal{O}_{d-1}$ localized at $x = 0$, with a form dictated by conformal invariance to be:

$$\mathcal{O}_d(\vec{y}, x) = \sum_n \frac{B^{\mathcal{O}_d}_{\mathcal{O}^n}}{x^{\Delta_d - \Delta_n}} \mathcal{O}^n_{d-1}(\vec{y}) \, . \tag{2.6}$$

The operators $\mathcal{O}^n_{d-1}$ appearing in this expansion can either be made from defect fields, or from reductions of ambient fields to the defect, or from both. For instance, if there are no interactions with the defect, (2.6) is simply the Taylor expansion of the ambient operator, and the operators $\mathcal{O}^n_{d-1}(\vec{y})$ are all of the form $\partial^n_x \mathcal{O}_d(\vec{y}, x)|_{x=0}$, of dimension $\Delta_n = \Delta_d + n$. Once we add interactions these Taylor modes can generally have anomalous dimensions and mix with defect operators. Note that in general a reduced operator like $[\partial^n_x \mathcal{O}_d](\vec{y}) \equiv \partial^n_x \mathcal{O}_d(\vec{y}, x)|_{x \to 0}$ cannot be defined without subtracting a divergence. The operators $\mathcal{O}^n_{d-1}(\vec{y})$ appearing in (2.6) are generally primaries (or descendants of primaries) of the defect conformal algebra; hence the natural way to get primaries from an ambient operator after the introduction of the defect is to compute its boundary operator product expansion.

One should note that in the free theory, the operators $[\partial^n_x \mathcal{O}_d](\vec{y})$ are not precisely the primaries, though there is a primary associated to each. This is obvious because the two-point functions of these operators do not vanish for different n's, even though the operators with different n have different scaling dimensions. The actual primaries are of the form

$$\left[\widetilde{\partial^n_x \mathcal{O}_d}\right] \equiv [\partial^n_x \mathcal{O}_d] + c_2 \nabla^2 \left[\partial^{n-2}_x \mathcal{O}_d\right] + c_4 (\nabla^2)^2 \left[\partial^{n-4}_x \mathcal{O}_d\right] + \cdots \, , \tag{2.7}$$

where ∇^2 is the $\vec{y}$-laplacian. Hence the BOPE expresses $\mathcal{O}_d$ in terms of both primaries and descendants of primaries. The constants c_i, which depend on the dimension Δ of $\mathcal{O}_d$, may be determined either by explicitly demanding annihilation by the special conformal generators, or by requiring orthogonality of the two-point functions. We will write some of them down in section 3.2.

As discussed in [27], we can insert the BOPE into equations such as (2.4), and use the results to obtain information about the dimensions of the operators living on the defect. The two-point function (2.4) has two different possible limits — small ξ and large ξ. The small-ξ limit corresponds to bringing the operators close together away from the defect, and it is natural to analyze this limit using the regular OPE of the ambient theory; the behavior of $f(\xi)$ in this limit is determined by the ambient OPE coefficients and by the one-point functions (2.2) of the ambient operators $\mathcal{O}^3_d$ appearing in the OPE of $\mathcal{O}^1_d$ and $\mathcal{O}^2_d$. On the other hand, the large-ξ limit corresponds to bringing the operators very close

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to the defect relative to their distance from one another. To analyze the behavior of $f(\xi)$ in this limit we plug the BOPE (2.6) into equation (2.4), for both operators, leading to

$$f(\xi) \simeq \sum_{n_1,n_2} B^{\mathcal{O}^1_d}_{\mathcal{O}^{n_1}} x^{\Delta_{n_1}} B^{\mathcal{O}^2_d}_{\mathcal{O}^{n_2}} (x')^{\Delta_{n_2}} \left\langle \mathcal{O}^{n_1}_{d-1}(\vec{y})\, \mathcal{O}^{n_2}_{d-1}(\vec{y}')\right\rangle . \tag{2.8}$$

In the limit where the operators are very close to the defect, we have $\xi \simeq (\vec{y}-\vec{y}')^2/(4xx')$. We see that the contribution from a primary operator $\mathcal{O}^n_{d-1}$ appearing in the BOPE[6] to the large-ξ behavior of $f(\xi)$ is $f(\xi) \simeq B^{\mathcal{O}^1_d}_{\mathcal{O}^n} B^{\mathcal{O}^2_d}_{\mathcal{O}^n} (4/\xi)^{\Delta_n}$. Contributions from descendant operators generally give higher powers of $1/\xi$ in the large-ξ limit. Thus, from the large-ξ behavior of (2.4) we can read off the dimensions (including the anomalous dimensions) of all the defect operators $\mathcal{O}^n_{d-1}$ appearing in the BOPE of both $\mathcal{O}^1_d$ and $\mathcal{O}^2_d$.

In the AdS/CFT correspondence, operators $\mathcal{O}_d(\vec{y},x)$ are mapped to fields living in the full AdS_{d+1} space, while operators $\mathcal{O}_{d-1}(\vec{y})$ are mapped either to fields living on an AdS_d brane or to the modes of the bulk fields (expanded near the brane). We can compute all the correlators described above also on this side of the correspondence, using string theory or using a supergravity approximation including a brane source (which is valid for large g_sN and small g_sM). We will perform some computations of this type in section 5.

2.3 Conserved currents

Symmetries in a dCFT come in two types. The simpler kind acts only on the defect variables. The associated currents are purely defect operators, and the holographic duals to these currents are gauge fields on the AdS_d brane. In the example of the D3/D5 system with M D5-branes, we have a global U(M) symmetry acting on the M hypermultiplets living on the defect, and the dual vector fields are the zero modes of the gauge fields living on the D5-branes. These currents are conserved in the usual way, and do not acquire anomalous dimensions.

The more interesting type of current corresponds to a symmetry acting on both ambient and defect fields. An example is the stress tensor, or the R-currents in the supersymmetric systems. Currents of this type may be written in the form

$$T_{\mu\nu}(\vec{y},x) = \mathcal{T}_{\mu\nu}(\vec{y},x) + \delta(x)\, \delta^k_\mu \delta^l_\nu\, t_{kl}(\vec{y}) , \tag{2.9}$$
$$J_\mu(\vec{y},x) = \mathcal{J}_\mu(\vec{y},x) + \delta(x)\, \delta^k_\mu\, j_k(\vec{y}) . \tag{2.10}$$

These currents are dual to fields moving in the AdS_{d+1} bulk, such as the graviton or graviphotons. The operators $\mathcal{J}$ and j (or $\mathcal{T}$ and t) are not separately conserved, but the total J obeys $\partial^\mu J_\mu = 0$ ($\partial^\mu T_{\mu\nu} = 0$) both away from the defect and on the defect. For the D3/D5 system, the fact that the energy-momentum tensor has the form (2.9) was noted in [11], and the $\mathrm{SU}(2)_H \times \mathrm{SU}(2)_V$ R-currents similarly have the structure (2.10).

In the free theory, $\mathcal{J}_\mu$ and j_k are separately conserved; j_k has dimension $\Delta = d-2$ and $\mathcal{J}_\mu$ has dimension $\Delta = d-1$. In the full theory, j_k is no longer conserved, and thus defect conformal invariance dictates that j_k must have an anomalous dimension, since a primary

[6] Normalizing the two-point function of $\mathcal{O}^n_{d-1}$ to be $\langle \mathcal{O}^n_{d-1}(\vec{y})\, \mathcal{O}^n_{d-1}(\vec{y}')\rangle = 1/[(\vec{y}-\vec{y}')^2]^{\Delta_n}$.

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vector operator of $\mathrm{SO}(d-1,2)$ can have dimension $\Delta = d-2$ if and only if it is a conserved current. A priori one might think that the appearance of such an anomalous dimension in correlation functions like $\langle JJ \rangle$ is inconsistent with the conservation of J. However, we will show in section 4 that the divergence in $\langle j\, j \rangle$ associated with the anomalous dimension is canceled in the full $\langle J\, J \rangle$ correlator by contributions from $\langle \mathcal{J}\, j \rangle$ and $\langle j\, \mathcal{J} \rangle$. Similar arguments apply to $T, \mathcal{T}$ and t, with the dimensions shifted up by one.

In the next section, we will discuss obtaining a BOPE for operators (including $T_{\mu\nu}$ and J_μ) from gravity considerations. Having done so, we will be in a position to put forward our proposal for a definition of local localization in a dCFT.

3. Local localization and the BOPE

In the presence of an AdS_d brane, it is natural to decompose AdS_{d+1} fields into modes transforming under the preserved AdS_d isometry group. This decomposition is analogous to the boundary operator product expansion that we performed in the previous section for field theory operators in the presence of the defect. In this section we will show that the mode decomposition and the BOPE are actually related by the AdS/CFT correspondence. This result will allow us to provide a precise proposal for the realization of local localization in dCFT. For convenience, we perform the analysis of this section in the Poincaré patch; the generalization to global AdS is straightforward.

3.1 BOPE as gravity mode decomposition

In studying the AdS/CFT correspondence, it is standard to represent AdS_{d+1} as a foliation of d-dimensional Minkowski spaces. For example, its metric may be written as

$$ds^2_{mink} = L^2 \left(e^{2\rho} \left\{ dx^2 + d\vec{y}^2 \right\} + d\rho^2 \right) , \tag{3.1}$$

where L is the curvature radius of AdS_{d+1} and each constant-ρ slice is a copy of Mink_d. On the other hand, in analyzing the bulk physics of an embedded AdS_d-brane, it is useful to express the geometry of AdS_{d+1} in terms of a foliation of AdS_d submanifolds,

$$ds^2_{ads} = L^2 \left(\cosh^2 r \left\{ e^{2w} d\vec{y}^2 + dw^2 \right\} + dr^2 \right) , \tag{3.2}$$

where each constant-r slice is an AdS_4, with $-\infty < r < \infty$ and the brane at $r = 0$. The coordinate systems are related by

$$x = e^{-w} \tanh r \,, \qquad e^{\rho} = e^{w} \cosh r \,, \tag{3.3}$$

and are depicted in figure 1. From here on we will set $L = 1$ (it can always be restored by dimensional analysis).

Every AdS_{d+1} bulk field $\phi_{d+1}(\vec{y}, w, r)$ of mass M, transforming in some representation of $\mathrm{SO}(d, 2)$, decomposes into a tower of AdS_d modes $\phi_{d,n}(\vec{y}, w)$ inhabiting representations of the preserved isometry group $\mathrm{SO}(d-1, 2)$. Each mode is multiplied by an appropriate wavefunction of the r-direction:

$$\phi_{d+1}(\vec{y}, w, r) = \sum_n \psi_n(r)\, \phi_{d,n}(\vec{y}, w) \,. \tag{3.4}$$

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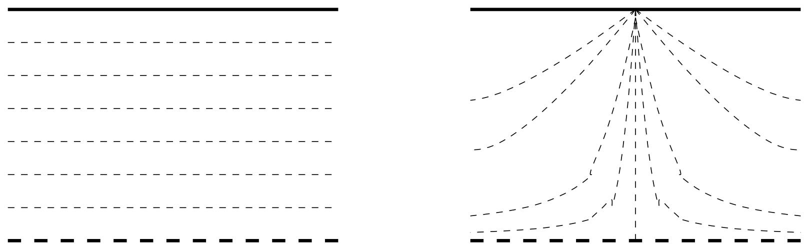

Figure 1: The two slicings of AdS_5. The horizontal axis is the direction x transverse to the brane and the vertical axis is the radial direction of AdS interpolating from the boundary (solid line) to the horizon (dashed line). The figure on the left shows lines of constant ρ while the figure on the right shows lines of constant r.

Among the data of the SO$(d-1,2)$ representation is an AdS_d-mass m_n for each $\phi_{d,n}$,

$$\partial_d^2 \phi_{d,n} = m_n^2 \phi_{d,n} \, , \tag{3.5}$$

where ∂_d^2 is the AdS_d-laplacian. The mass m_n and the wavefunction $\psi_n(r)$ may be determined by solving the wave equation for $\phi_{d+1}(\vec{y}, x, r)$. In general the backreaction of the brane may produce a more general warp factor $A(r)$, $ds^2 = dr^2 + e^{2A(r)} ds^2_{AdS_d}$, although (3.2) will continue to hold at large $|r|$; this more general metric still preserves AdS_d isometries associated with dual dCFT. To linear order the wave equation then reduces to an ordinary differential equation for the wavefunction $\psi_n(r)$,

$$\partial_r^2 \psi_n(r) + dA'(r)\partial_r \psi_n(r) + e^{-2A(r)} m_n^2 \psi_n(r) - M^2 \psi_n(r) = 0 \, . \tag{3.6}$$

This will receive corrections from various interactions in the brane worldvolume theory,[7] all of which affect the calculation of the masses m_n.

The field ϕ_{d+1} of mass M is dual to an ambient operator $\mathcal{O}_d(\vec{y}, x)$ of dimension Δ_d (with $\Delta_d(\Delta_d - d) = M^2$) in the dCFT. Analogously, since the $\phi_{d,n}$ inhabit an effective AdS_d theory (they are representations of SO$(d-1,2)$), they are related to dual "defect operators" $\mathcal{O}_{d-1,n}(\vec{y})$. We wish to interpret the bulk relation (3.4) in terms of some operator decomposition of $\mathcal{O}_d$ into the $\mathcal{O}_{d-1,n}$. In view of the discussion in the previous section, it is natural to identify the operators $\mathcal{O}_{d-1,n}(\vec{y})$ with the reduced operators arising from the expansion of $\mathcal{O}_d$ near the defect.

In the AdS/CFT correspondence, the relation between bulk fields and the values of operators in the "boundary" theory proceeds by choosing a function f on AdS_{d+1} that has a pole on the boundary, and multiplying each field by a power of this function determined by the mass of the field to obtain finite boundary values which are related to the dual operators [25]. The freedom to pick this function corresponds to the freedom to make conformal transformations on the boundary theory; in particular the choice of f determines the metric on the boundary.

[7]The brane interactions will generally cause a mixing between the modes corresponding to different bulk fields ϕ_{d+1}, though we neglect this here. However, precisely the same phenomenon occurs also in the BOPE, and it is easy to generalize our discussion to incorporate it.

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The expectation values of normalizable solutions for ϕ_{d+1} compute in this fashion vacuum expectation values of the dual operators $\mathcal{O}_d$. One possible choice of f, motivated by (3.2), is to pick the function f to be the AdS_d-slicing warp factor, $\cosh r$. For this choice we obtain the relation

$$\phi_{d+1}(\vec{y},w,r) = \frac{(\cosh r)^{-\Delta_d}}{2\Delta_d - d} \langle \mathcal{O}_d(\vec{y},w)\rangle + \mathcal{O}\left((\cosh r)^{-(\Delta_d+2)}\right), \tag{3.7}$$

near the boundary of AdS_{d+1} at $r=\pm\infty$. However, the metric obtained on the boundary from this choice of f will be induced from the AdS_d slicing, and hence will be two copies of AdS_d, glued together along their common boundary, which is the location of the defect.

This is not the boundary metric we desire, as we are interested in the dCFT living on flat space (where our analysis of the previous section applies). The choice of function with pole f that accomplishes this is of course the warp factor for the Minkowski foliation, e^ρ. This will induce a flat metric, and we shall see that it also induces a nontrivial spatial variation transverse to the defect in the boundary values of the wavefunctions $\psi_n(r)$. In our choice of the flat boundary metric the relation between the bulk field and the expectation value of the dual operator in the dCFT is hence

$$\phi_{d+1}(\vec{y},x,\rho) = \frac{e^{-\Delta_d \rho}}{2\Delta_d - d} \langle \mathcal{O}_d(\vec{y},x)\rangle + \mathcal{O}\left(e^{-(\Delta_d+2)\rho}\right), \tag{3.8}$$

as $\rho\to\infty$. As usual, in the absence of a source term for the field (which would be larger than the terms in (3.8) for $\Delta_d \geq d/2$), the coefficient of the leading term is identified with the VEV of the operator [28]. We want to read this as an operator equation, since it should still be true in the presence of sources, as long as you are not on top of the source – in the presence of insertions of other operators one re-solves for the perturbed gravity background, and again extracts the coefficient of the same term as the value of the operator.

Now, ϕ_{d+1} also obeys the expression (3.4). Consequently, using the coordinate relations (3.3) evaluated near the boundary, we may write

$$\langle \mathcal{O}_d(\vec{y},x)\rangle = \lim_{\rho\to\infty} e^{\Delta_d \rho}(2\Delta_d - d)\sum_n \psi_n(r)\,\phi_{d,n}(\vec{y},w)\Big|_{e^r=2xe^\rho,\, e^{-w}=x}. \tag{3.9}$$

The wavefunctions $\psi_n(r)$ have a universal scaling near the boundary

$$\psi_n(r) = C_n (e^r)^{-\Delta_d} + \mathcal{O}\left(e^{-(\Delta_d+2)r}\right), \tag{3.10}$$

regardless of n. This is because for large $|r|$ the geometry reverts to AdS_{d+1} (3.2) and $e^{-2A(r)} \simeq \cosh^{-2} r$ vanishes, meaning all modes are governed by

$$\partial_r^2 \psi_n + dA'(r)\partial_r\psi_n - M^2\psi_n = 0\,, \tag{3.11}$$

leading to the result (3.10). The expression (3.9) then becomes

$$\langle \mathcal{O}_d(\vec{y},x)\rangle = (2x)^{-\Delta_d}(2\Delta_d - d)\sum_n C_n\,\phi_{d,n}(\vec{y},w)\Big|_{e^{-w}=x}, \tag{3.12}$$

with some constants C_n determined by the solution for $\psi_n(r)$.

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The fields $\phi_{d,n}$ live in an AdS_d space, and by the basic principle of AdS/dCFT they are related to $(d-1)$-dimensional operators $\mathcal{O}_{d-1,n}(\vec{y})$ living on the intersection of the AdS_d brane with the boundary. The $\mathcal{O}_{d-1,n}$ have conformal dimensions Δ_n determined by the $\phi_{d,n}$ masses as $\Delta_n(\Delta_n - d + 1) = m_n^2$, and obey a relation completely analogous to (3.8):

$$\phi_{d,n}(\vec{y}, w) \simeq \frac{e^{-\Delta_n w}}{2\Delta_n - d + 1} \langle \mathcal{O}_{d-1,n}(\vec{y}) \rangle + \mathcal{O}\left(e^{-(\Delta_n+2)w}\right) \tag{3.13}$$

at large w. However, there is an essential difference between the two equations for the purposes of our derivation. While the coordinate e^ρ in (3.8) plays a purely holographic role, and is eliminated in the process of computing boundary data, the coordinate e^w appearing in (3.13) is identified with $1/x$, which survives in the field theory. This is a consequence of our order of limits, going to the boundary $\rho \to \infty$ before taking $x \to 0$.

As a result, the subleading terms in (3.13) all survive in the boundary limit. Using the AdS_d wave equation

$$(\partial_w^2 + (d-1)\partial_w + e^{-2w}\nabla^2 - m_n^2)\phi_{d,n}(\vec{y}, w) = 0\,, \tag{3.14}$$

where ∇^2 is again the laplacian in the $\vec{y}$ directions, they may be calculated recursively in terms of derivatives ∇^2 of the VEV $\langle \mathcal{O}_{d-1,n}(\vec{y}) \rangle$:

$$\phi_{d,n}(\vec{y}, w) \;=\; e^{-\Delta_n w} \sum_{k=0}^{\infty} a_{k,n}\, e^{-2kw}\, \nabla^{2k} \langle \mathcal{O}_{d-1,n}(\vec{y}) \rangle\,, \tag{3.15}$$

$$a_{k-1,n} \;=\; -\left[(\Delta_n + 2k)^2 - (\Delta_n + 2k)(d-1) - m_n^2\right] a_{k,n}\,, \qquad a_{k=0,n} = \frac{1}{2\Delta_n - d + 1}\,.$$

Hence we have

$$\langle \mathcal{O}_d(\vec{y}, x) \rangle = (2x)^{-\Delta_d}(2\Delta_d - d) \sum_n C_n \sum_k a_{k,n}\, x^{\Delta_n + 2k}\, \nabla^{2k} \langle \mathcal{O}_{d-1,n}(\vec{y}) \rangle\,. \tag{3.16}$$

Extrapolating this to an operator relation, we obtain precisely a boundary operator product expansion of the form

$$\mathcal{O}_d(\vec{y}, x) = \sum_i \frac{B^{\mathcal{O}_d}_{\mathcal{O}_i}}{x^{\Delta_d - \Delta_i}} \mathcal{O}_{d-1,i}(\vec{y})\,, \tag{3.17}$$

where i runs over all values of n and k, with the $\mathcal{O}_i(\vec{y})$ corresponding to all $\nabla^{2k}\mathcal{O}_n(\vec{y})$, and with $\Delta_i = \Delta_n + 2k$.

It is quite satisfying to derive the BOPE decomposition directly from gravity considerations. The analysis here was for scalar operators/fields, but it can easily be generalized also to other fields, and in particular to symmetric tensors which are relevant for the localization of gravity. We see directly that the BOPE of a primary d-dimensional operator $\mathcal{O}_d(\vec{y}, x)$ will generically contain both $d-1$-dimensional primaries $\mathcal{O}_{d-1,n}(\vec{y})$ and their descendants $\nabla^{2k}\mathcal{O}_{d-1,n}(\vec{y})$. Moreover, the relation (3.13) establishes the one-to-one correspondence between modes ϕ_d in the AdS decomposition (3.4) and the primaries $\mathcal{O}_{d-1,n}$ appearing in the BOPE. Consequently, we expect locally localized modes to be associated to particular reduced primary operators appearing in the BOPE.

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3.2 Example of the no-brane case

It is illuminating to examine (3.16) in the elementary case of a "phantom" brane. This means that we study fields on AdS_{d+1} without including any interactions with a brane, but we proceed with the decomposition into AdS_d modes regardless. As discussed in section 2, for this trivial case the BOPE is simply a Taylor expansion in x,

$$\mathcal{O}(\vec{y},x) = [\mathcal{O}](\vec{y}) + x[\partial_x\mathcal{O}](\vec{y}) + \frac{x^2}{2!}[\partial_x^2\mathcal{O}](\vec{y}) + \frac{x^3}{3!}[\partial_x^3\mathcal{O}](\vec{y}) + \cdots . \tag{3.18}$$

We need to bear in mind that the $[\partial_x^n\mathcal{O}]$ are generically not primary operators for $n > 1$, but rather each corresponds to the sum of a primary and descendants. For our explicit comparison, we choose to study the $d = 4$ case, for which the first few primaries $\widetilde{[\partial_x^n\mathcal{O}]}$ at $n > 1$ are

$$\begin{aligned}\widetilde{[\partial_x^2\mathcal{O}]} &\equiv \left[\partial_x^2\mathcal{O} + \frac{1}{2\Delta-1}\nabla^2\mathcal{O}\right], \qquad \widetilde{[\partial_x^3\mathcal{O}]} \equiv \left[\partial_x^3\mathcal{O} + \frac{3}{2\Delta+1}\nabla^2\partial_x\mathcal{O}\right], \\ \widetilde{[\partial_x^4\mathcal{O}]} &\equiv \left[\partial_x^4\mathcal{O} + \frac{6}{2\Delta+3}\nabla^2\partial_x^2\mathcal{O} + \frac{3}{(2\Delta+1)(2\Delta+3)}\nabla^2\nabla^2\mathcal{O}\right],\end{aligned} \tag{3.19}$$

where $\mathcal{O}$ has conformal dimension Δ. In terms of these primaries and their descendants we can rewrite the Taylor expansion BOPE in the form

$$\begin{aligned}\mathcal{O}(\vec{y},x) &= [\mathcal{O}](\vec{y}) + x[\partial_x\mathcal{O}](\vec{y}) + \frac{x^2}{2}\widetilde{[\partial_x^2\mathcal{O}]}(\vec{y}) + \frac{x^3}{6}\widetilde{[\partial_x^3\mathcal{O}]}(\vec{y}) - \\ &\quad - \frac{x^2}{2(2\Delta-1)}\nabla^2[\mathcal{O}](\vec{y}) - \frac{x^3}{2(2\Delta+1)}\nabla^2[\partial_x\mathcal{O}](\vec{y}) + \cdots .\end{aligned} \tag{3.20}$$

We seek to derive this expansion from the gravity relation (3.16).

Let us now restrict ourselves further to the case of $d = \Delta = 4$. For this case the wavefunctions for a "phantom brane" were calculated in [7, 29], and found to be

$$\begin{aligned}\psi_n(r) &= (\cosh^{-4} r)\; {}_2F_1\left(\frac{5}{2}+\frac{n}{2}, -\frac{n}{2}; 3; \cosh^{-2} r\right) \\ &\sim e^{-4\rho}\frac{1}{x^4},\end{aligned} \tag{3.21}$$

where $n = 0, 1, 2, \ldots$ and $m_n^2 = (n+1)(n+4)$. We consequently have $C_n = 16$ in this case, and we obtain from (3.12)

$$\langle\mathcal{O}(\vec{y},x)\rangle = 4x^{-4}\sum_n \phi_{d=4,n}(\vec{y},w)\Big|_{e^{-w}=x}, \tag{3.22}$$

where we set $\mathcal{O}_{d=4}(\vec{y},x) \equiv \mathcal{O}(\vec{y},x)$. Let us first consider the primaries, which come from the leading term in each $\phi_{d=4,n}$, as in (3.13). The mass $m_n^2 = (n+1)(n+4)$ implies $\Delta_n = 4 + n$, giving us from (3.13)

$$\phi_{d=4,n}(\vec{y},w) = \frac{x^{4+n}}{2n+5}\langle\mathcal{O}_n(\vec{y})\rangle + \cdots, \tag{3.23}$$

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with $\mathcal{O}_{d-1=3,n}(\vec{y}) \equiv \mathcal{O}_n(\vec{y})$. Since the $\mathcal{O}_n$ are primaries, they must correspond to the $\widetilde{[\partial_x^n \mathcal{O}]}(\vec{y})$ up to a possible normalization, $\mathcal{O}_n(\vec{y}) \equiv \beta_n \, \widetilde{[\partial_x^n \mathcal{O}]}(\vec{y})$, leading to the primary part of the BOPE

$$\mathcal{O}(\vec{y}, x) = \sum_n \frac{4\beta_n}{2n+5} \, x^n \, \widetilde{[\partial_x^n \mathcal{O}]}(\vec{y}) \; + \text{descendants} \, . \tag{3.24}$$

For this to match (3.20) we must have $4\beta_n/(2n+5) = 1/n!$. In order to verify this we need to compare the normalization of the $\mathcal{O}_n(\vec{y})$ to that of the $\widetilde{[\partial_x^n \mathcal{O}]}(\vec{y})$. A canonically normalized AdS_d scalar with dimension Δ gives rise to a dual operator with the two-point function

$$\langle \mathcal{O}(\vec{y}) \, \mathcal{O}(0) \rangle = \frac{1}{\pi^{(d-1)/2}} (2\Delta - d + 1) \frac{\Gamma(\Delta)}{\Gamma(\Delta - \frac{d-1}{2})} \frac{1}{y^{2\Delta}} \, . \tag{3.25}$$

However, the $\phi_{d,n}$ are in general not canonically normalized; their normalization in the action is determined by dimensionally reducing the AdS_{d+1} action for ϕ_{d+1}, and this leads to an extra factor of $\int dr e^{2A(r)} \psi_n(r)^2$ outside the kinetic terms for $\phi_{d,n}$, which then appears in the denominator of the two-point function. For the case of the $d = \Delta = 4$ phantom brane, we can compute this by integrating (3.21) to obtain

$$\langle \mathcal{O}_n(\vec{y}) \, \mathcal{O}_n(0) \rangle = \frac{(2n+5)(n+4)!}{128 \, n!} \times \frac{1}{\pi^{3/2}} (2n+5) \frac{\Gamma(n+4)}{\Gamma(n+\frac{5}{2})} \frac{1}{y^{8+2n}} \, . \tag{3.26}$$

Thus, the coefficients of the BOPE only come out right if it happens that

$$\begin{aligned} \langle \widetilde{[\partial_x^n \mathcal{O}_n]}(\vec{y}) \, \widetilde{[\partial_x^n \mathcal{O}_n]}(0) \rangle &= \frac{1}{\beta_n^2} \langle \mathcal{O}_n(\vec{y}) \, \mathcal{O}_n(0) \rangle \\ &= \frac{n!(n+3)!(n+4)!}{8\pi^{3/2} \Gamma(n+\frac{5}{2})} \frac{1}{y^{8+2n}} \, . \end{aligned} \tag{3.27}$$

We have not been able to compute the two-point functions of the primaries for general n, but we have checked that the first 4 primaries listed in (3.19) do satisfy (3.27) exactly, assuming that the 4D $\mathcal{O}(y, x)$ has the canonical two-point function given as the AdS_{d+1} version of (3.25) with $d = \Delta = 4$. Hence the Taylor series structure is exactly reproduced for the primaries up to this order. We may regard (3.27) as a prediction for the norms of the higher primaries.

One is then left needing to match the descendants; the terms $a_{k,n}$ with $k \neq 0$ in the expansion (3.15) must produce the second line of (3.20). We have verified that this is indeed the case up to $\mathcal{O}(x^4)$. The $\mathcal{O}(x^4)$ term is the most intricate, as the Taylor term $[\partial_x^4 \mathcal{O}]$ is composed of a combination of the primary $\widetilde{[\partial_x^4 \mathcal{O}]}$ as well as the $k = 1$ descendant of $\widetilde{[\partial_x^2 \mathcal{O}]}$ and the $k = 2$ descendant of $[\mathcal{O}]$. Higher terms will be more complicated still.

To summarize this subsection, we have verified explicitly to fourth order in x that the mode decomposition in the presence of the phantom brane is precisely equivalent to a simple Taylor series for the dual field theory operator for the case $d = \Delta = 4$. We expect this agreement to persist to all orders, and to pertain also to other cases. In particular, when there is a genuine interacting brane, we expect (3.16) to produce the reduced operators in the full interacting BOPE for the dual dCFT.

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3.3 Local localization in dCFT

Local localization occurs when one of the modes in the decomposition of the graviton,

$$h_{kl}(\vec{y}, r, w) = \sum_n \psi_n(r)\, (h_{kl})_{d,n}(\vec{y}, w)\,, \tag{3.28}$$

has an AdS_d mass that is hierarchically small compared to the scale of the AdS curvature, which characterizes the masses of the other fields. For the simplest system of a gravitating 4D brane coupled to 5D gravity with a cosmological constant [7], the existence of the locally localized mode has been confirmed both numerically [7, 30] and analytically [31, 29]. One may expect that local localization appears in many other gravitating systems as well. One way to try to confirm this is to explicitly construct such systems, in string theory for example; this has thus far proven difficult (see [23] for the state of the art in the D3/D5 system). Since every gravity phenomenon should have a field theory interpretation, even novel ones like local localization, another way to proceed is to abstract the concept to an effect in the dual field theory, and then look for local localization in the dCFT duals even when the exact gravity solution including brane backreaction is not known. Understanding the nature of local localization in the dCFT dual is also interesting in its own right, and this is what we attempt to do here.

The analysis of subsection 3.1 can be readily adapted to the study of the graviton, as the transverse polarizations obey a massless scalar wave equation. It is easy to verify that the wavefunctions discussed above are localized near the brane if and only if $\Delta_n < \Delta_d$; in particular near the boundary the wavefunctions behave as $x^{\Delta_n - \Delta_d}$. Thus, we can say that a mode $\phi_{d,n}$ begins to be localized around the brane whenever its mass is such that the associated scaling dimension Δ_n is smaller than the scaling dimension Δ_d associated with the parent field ϕ_{d+1}.

Through the equivalence of the AdS mode expansion and the BOPE that we have established, and by using the usual AdS/CFT relation between mass and conformal dimension, we can naturally recast this result in a field theory language as a relation between anomalous dimensions. In particular, when local localization of gravity occurs, a reduced operator in the BOPE expansion of the d-dimensional stress tensor has a conformal dimension approaching $d-1$. As the dimension becomes closer to $d-1$, gravity will be localized to the brane over a larger and larger range of length scales. Note that a dimension $\Delta_n < d-1$ is forbidden by unitarity, and $\Delta_n = d-1$ can occur only if the reduced stress tensor is conserved on its own and the theory has two decoupled sectors.

We can therefore make a natural definition:

> *Local localization of gravity occurs in a defect conformal field theory when there is an operator in the BOPE of the stress tensor with* $\Delta - (d-1) \ll 1$.

In more generality, one can abstract the notion to a generic localized field in a dCFT; the corresponding operator should have in its BOPE a defect operator with a conformal dimension smaller than that of the parent d-dimensional operator.

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Obviously, local localization does not occur when we have no interactions with the defect. This is evident since the BOPE in that case is just the Taylor expansion, and the conformal dimensions of the reduced operators are simply $\Delta_d + n$, $n = 0, 1, 2, \ldots$, none of which is smaller than Δ_d. As the coupling is turned on, localization will occur if the $n = 0$ reduced operator acquires a negative anomalous dimension. This is another example of a familiar property of AdS/CFT, that phenomena in classical gravity can be mapped to quantum effects in the large-coupling limit of the dual field theory.

Our discussion in this section so far was limited only to the "reduced operators" arising as modes of the ambient operators. At first sight, it seems that another possible candidate spin-2 operator to have a scaling dimension close to $\Delta = d - 1$ is the defect-localized part of the stress tensor t_{kl} (2.9). In the free theory, its dimension is exactly $\Delta = d - 1$, and since it is a part of a conserved current, one may be tempted to believe its dimension is not renormalized and remains $d - 1$.

This point of view is problematic, however. The defect part of the stress tensor is a fundamentally defect operator, not a moment of an ambient one; consequently it is expected to be dual to dynamics localized on the AdS_d brane itself, rather than to bulk physics. In the well-understood probe regime of the D3/D5 system, with $g_s N$ large and $g_s M$ small, t_{kl} is therefore dual to a spin-2 open string field localized on the D5-branes. Since there are no massless spin-2 open string modes, the dual of t_{kl} must be a massive stringy mode, and thus it should acquire a large anomalous dimension $\Delta \simeq (g_s N)^{1/4}$. On the other hand, the dimensions of the BOPE operators coming from $\mathcal{T}_{\mu\nu}$ are still close to $\Delta^{(n)} = 4 + n$ in this regime, since the backreaction of the brane is small. Hence the "naturally localized" field t_{kl} is not a part of the onset of locally localized gravity in the small-$g_s M$ regime. Similarly, the defect parts j_k of the $\mathrm{SU}(2)_H \times \mathrm{SU}(2)_V$ R-symmetry currents (2.10) should also acquire anomalous dimensions in the D3/D5 system, as they sit in the same multiplet as the energy-momentum tensor.

In more general systems, we expect the story to be similar. There is no example in string theory of a light spin-2 brane mode that could be dual to t_{kl}, and hence we expect the dual field to generically be heavy in regimes where a brane description is adequate. In section 4 we demonstrate that the defect component of a current indeed acquires an anomalous dimension in a simple dQFT, and we show how this is consistent with the conservation of the full current. Similar computations could also be done for the stress tensor.

In section 5, we perform the calculations necessary for the supergravity computation of the anomalous dimensions of reduced scalar operators, to leading order in $g_s M$. We shall see that anomalous dimensions generically appear here as well, but since they are corrections of order $g_s M$, they are not large in the probe limit. We expect that similar computations will give the anomalous dimensions of the BOPE modes of T; in the D3/D5 system these anomalous dimensions are related by supersymmetry to those of some reduced scalar operators, so the computation of the latter suffices for the analysis of locally localized gravity. Such computations should reveal whether there is an onset of localization as the backreaction of the brane starts to be taken into account.

The hope for a truly localized graviton lies, of course, with strong backreaction, that is large $g_s M$ as well as large $g_s N$. In this regime we do not know how to compute the

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anomalous dimensions of the various fields, and we do not know whether the dimension of t is smaller or larger than that of the BOPE modes coming from $\mathcal{T}$. Since we show in this paper that both t and the modes coming from $\mathcal{T}$ have anomalous dimensions, it is possible that one of these operators has a dimension such that $\Delta - (d-1) \ll 1$ at large $g_s M$, so that localized gravity would be realized in string theory in this regime. It is hard to say whether t could come down in dimension at large $g_s M$ and participate in the local localization, or if this localization necessarily involves only a mode of $\mathcal{T}$. For large $g_s M$ there could in any case be a large mixing between t and the modes of $\mathcal{T}$, so the distinction may well be meaningless.

4. Anomalous dimensions of defect operators

In this section we wish to verify that the picture we have developed, in which the $(d-1)$-dimensional component of the stress tensor acquires an anomalous dimension despite the fact that the full stress tensor containing it is conserved, is consistent. For simplicity, rather than analyzing the stress tensor in the D3/D5 system, we pick a toy model involving a subset of the fields of the D3/D5 system, and we examine the same phenomenon for a conserved current in this toy model. We expect the behavior of the stress tensor itself (which, in the D3/D5 system, is related by supersymmetry to the currents) to be analogous.

As our toy model, we consider an elementary dQFT (not a conformal theory) given by the lorentzian action

$$S_{dQFT} = \int d^4 z \left[-\frac{i}{4}\bar{\lambda}^{mi}\gamma^\mu \partial_\mu \lambda^{im} + \delta(x)\left(-i\bar{\Psi}^i \rho^k \partial_k \Psi^i + \partial^k \bar{q}^m \partial_k q^m \right) + \right.$$
$$\left. + g\delta(x)\left(i\bar{q}^m (\bar{\lambda}_1)^{mi}\Psi^i - i\bar{\Psi}^i (\lambda_1)^{im} q^m \right) \right].$$

Here q is a complex scalar, Ψ is a complex 3D spinor and λ is a 4D Majorana spinor; λ_1 is the projection of λ onto two of its components, see [11]. In this section we use z^μ as a shorthand for (y^k, x). The theory (4.1) is a subset of the dynamics of the D3/D5 dCFT [11], keeping only the fermions in the ambient four-dimensional theory. The $\mathrm{SU}(2)_V \times \mathrm{SU}(2)_H$ currents associated to the global symmetry (which is the R-symmetry in the full dCFT) are

$$\begin{aligned} J_V^{A\mu} &= \frac{1}{4}\bar{\lambda}^{mi}\gamma^\mu (T^A)_{ij}\lambda^{jm} + \delta(x)\,\delta_k^\mu\,\bar{\Psi}^i \rho^k (T^A)_{ij}\Psi^j\,, \\ J_H^{I\mu} &= -\frac{1}{4}\bar{\lambda}^{mi}\gamma^\mu \lambda^{in}(T^I)_{nm} - i\delta(x)\,\delta_k^\mu\,\bar{q}^m \overleftrightarrow{\partial^k}(T^I)_{mn} q^m\,, \end{aligned} \tag{4.1}$$

which are of the form (2.10). The matrices T^A and T^I here are the generators of the global symmetry group; the rest of the notations are explained in the appendix. The $\mathrm{SU}(2)_H$ indices m, n will play no role in our computation in this section, but we keep them so that (4.1) is a subtheory of the D3/D5 dCFT.

We shall study the two-point function of J_V^μ to two-loop order, and we drop the subscript V henceforth. We work directly in coordinate space and resolve divergences using differential regularization [32]. We are interested in properties of the correlator $\langle J(z_1) J(z_2)\rangle$

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for separated points, so only subdivergent parts of 2-loop diagrams are relevant. We will demonstrate that the defect current j is not protected, and we compute its anomalous dimension. The correlator $\langle j(y_1)\, j(y_2)\rangle$ thus depends on the renormalization scale M. We will show that the scale-dependence cancels in the full correlator $\langle J(z_1)\, J(z_2)\rangle$, consistent with the conservation of J. This comes about because $\langle \mathcal{J}\, j\rangle$ contains a subdivergence localized on the defect, leading to a scale-dependent contribution which provides the needed cancellation. To the order considered here $\langle \mathcal{J}\, \mathcal{J}\rangle$ is UV finite and does not participate in the cancellation.

To perform the computations, we find it convenient to employ a 4D spinor notation. A 4D spinor index decomposes into a 3D spinor index $\alpha = 1, 2$ and an internal index $a = 1, 2$; only the $a = 1$ component of λ participates in the defect interaction. Define the 4D spinor

$$\Psi^i_{\beta b} \equiv \Psi^i_\beta \delta_{1b}\,. \tag{4.2}$$

The interaction in the lagrangian (4.1) then becomes

$$S_{dQFT} \supset \int d^4z\, g\delta(x) \left(i\bar{q}^m \bar{\lambda}^{mi}\Psi^i - i\bar{\Psi}^i\lambda^{im}q^m\right)\,, \tag{4.3}$$

where λ and Ψ are both 4D spinors. The propagator of the 4D Ψ then contains the projection matrix $P_+ \equiv (1 + \gamma^5\gamma^x)/2$, as detailed in the appendix.

We study the order g^2 contributions to the correlator

$$\begin{aligned}\langle J^{Ak}(z_1)\, J^{Bl}(z_2)\rangle \equiv &\left\langle \mathcal{J}^{Ak}(z_1)\, \mathcal{J}^{Bl}(z_2)\right\rangle + \delta(x_2)\left\langle \mathcal{J}^{Ak}(z_1)\, j^{Bl}(y_2)\right\rangle + \\ &+\delta(x_1)\left\langle j^{Ak}(y_1)\, \mathcal{J}^{Bl}(z_2)\right\rangle + \delta(x_1)\delta(x_2)\left\langle j^{Ak}(y_1)\, j^{Bl}(y_2)\right\rangle.\end{aligned}$$

Here, A and B are indices in the adjoint of $\mathrm{SU}(2)_V$, and k, l are $d = 3$ vector indices. Figures 2, 3 and 4 show the lowest-order diagrams involving interactions that contribute to correlators of $\langle jj\rangle$, $\langle \mathcal{J}j\rangle$ and $\langle \mathcal{J}\mathcal{J}\rangle$, respectively. In the figures we use dashed lines for scalars and unbroken lines for fermions, thick lines for ambient fields and thin lines for defect fields. The filled dots correspond to insertions of j and the open dots to insertions of $\mathcal{J}$. All results are stated for the euclidean continuation of the correlation functions.

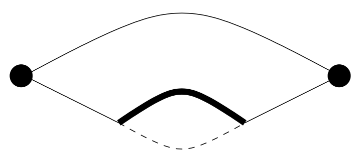

Figure 2: The leading diagram contributing to the $\langle jj\rangle$ current-current correlator.

In the $\langle j\, j\rangle$ diagram of figure 2 we have a Ψ loop where one Ψ propagator contains a λ/q self-energy loop. The result is

$$\left\langle j^{Ak}(y_1)\, j^{Bl}(y_2)\right\rangle = -8\, g^2\, \mathrm{Tr}\left(T^A T^B\right) \int d^3y_3\, d^3y_4\, \delta_{34}\, \mathrm{Tr}\left[\gamma^k\, \hat{s}_{12}\, \gamma^l\, \hat{s}_{23}\, S_{34}\, \hat{s}_{41}\right], \tag{4.4}$$

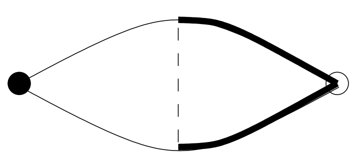

Figure 3: The leading diagram contributing to the $\langle \mathcal{J}j\rangle$ current-current correlator.

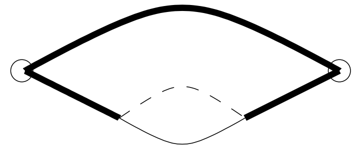

Figure 4: The leading diagram contributing to the $\langle \mathcal{J}\mathcal{J}\rangle$ current-current correlator.

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where S_{34} denotes the fermion propagator from $z_3 = (\vec{y_3}, x = 0)$ to z_4, and similarly for the other propagators; see the appendix. The coefficient comes from a 2 for the two diagrams, a 2 for the $\mathrm{SU}(2)_H$-index loop and a 2 for the index reduction of the λ propagator.

In $\langle \mathcal{J} j \rangle$, we have an exchange diagram where a $\Psi\Psi q$ triangle and a $\lambda\lambda q$ triangle share the q propagator, drawn in figure 3. We find

$$\left\langle \mathcal{J}^{Ak}(z_1)\, j^{Bl}(y_2) \right\rangle = -4\, g^2\, \mathrm{Tr}\left(T^A T^B\right) \int d^3y_3\, d^3y_4\, \delta_{34}\, \mathrm{Tr}\left[S_{31}\, \gamma^k\, S_{14}\, \hat{s}_{42}\, \gamma^l\, \hat{s}_{23}\right]. \quad (4.5)$$

$\langle j\, \mathcal{J} \rangle$ follows directly. The numerical factor includes 1/4 for the $\mathcal{J}$ current, 2 for two different Majorana contractions, 2^2 for the $\mathrm{SU}(2) \times \mathrm{SU}(2)$ index reduction in the two λ propagators as in (A.3), and 2 for the $\mathrm{SU}(2)_H$-index loop.

In $\langle \mathcal{J}\mathcal{J} \rangle$, the leading diagram is the λ loop with a self-energy interaction, drawn in figure 4. Due to the Majorana condition there are 8 distinct Wick contractions, which in the end each contribute equally to the amplitude

$$\left\langle \mathcal{J}^{Ak}(z_1)\, \mathcal{J}^{Bl}(z_2) \right\rangle = -8\, g^2\, \mathrm{Tr}\left(T^A T^B\right) \int d^3y_3\, d^3y_4\, \delta_{34}\, \mathrm{Tr}\left[S_{41}\, \gamma^k\, S_{12}\, \gamma^l\, S_{23}\, \hat{s}_{34}\right]. \quad (4.6)$$

The factor of 8 can be understood as follows: $(1/4)^2$ for the definition of the two currents, 8 for the eight separate terms, 2^3 for the $\mathrm{SU}(2) \times \mathrm{SU}(2)$ index reduction in the three λ propagators as in (A.3), and 2 for the $\mathrm{SU}(2)_H$-index loop.

We now proceed to the evaluation and regularization of these diagrams. The first point to notice is that there is no subdivergence in $\langle \mathcal{J}\mathcal{J} \rangle$, and the y_3, y_4 integrals converge for separated external points.

Next we analyze $\langle jj \rangle$. In detail the amplitude is

$$\begin{aligned} \langle j^{Ak}(y_1)\, j^{Bl}(y_2) \rangle = & -\frac{8\, g^2}{(4\pi^2)(4\pi)^4}\, \mathrm{Tr}\left(T^A T^B\right) \mathrm{Tr}[P_+ \gamma^k \gamma^m \gamma^l \gamma^n \gamma^p \gamma^q] \times \\ & \times \int d^3y_3\, d^3y_4\, \frac{1}{|y_{34}|}\, \partial_m \frac{1}{|y_{12}|}\, \partial_n \frac{1}{|y_{23}|}\, \partial_p \frac{1}{|y_{34}|^2}\, \partial_q \frac{1}{|y_{41}|}\,. \end{aligned} \quad (4.7)$$

Using differential regularization we first confront the subdivergence which occurs in the λ/q loop. We start by writing

$$\frac{1}{|y_{34}|}\, \partial_p \frac{1}{|y_{34}|^2} = \frac{2}{3} \partial_p \frac{1}{|y_{34}|^3}\,. \quad (4.8)$$

This would lead to a divergence in the integrals over y_3 and y_4. In differential regularization this is handled by means of the substitution

$$\frac{1}{|y_{34}|^3} = -\partial_p \nabla^2 \frac{\log M|y_{34}|}{|y_{34}|}\,, \quad (4.9)$$

where ∇^2 is the 3D laplacian, and M is an arbitrary mass unit. It is straightforward to show that, after insertion of (4.8), (4.9), the integrals in (4.7) can be done using the delta functions which appear after partial integration. For our purposes, it is sufficient to focus

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on the divergent (M-dependent) part. To isolate this we apply $M\partial/\partial M$. The regulated subdivergence (4.8) (which contains all the M-dependence) then becomes

$$-\frac{2}{3}\partial_p\nabla^2\frac{1}{|y_{34}|} = \frac{2}{3}\,(4\pi)\,\partial_p\delta(y_{34})\,. \tag{4.10}$$

Substituting back into $\langle jj\rangle$, we find

$$\begin{aligned} M\frac{\partial}{\partial M}\left\langle j^{Ak}(y_1)\,j^{Bl}(y_2)\right\rangle &= -\frac{16\,g^2}{3(4\pi^2)(4\pi)^3}\,\mathrm{Tr}\left(T^AT^B\right)\,\mathrm{Tr}[P_+\gamma^k\gamma^m\gamma^l\gamma^n\gamma^p\gamma^q]\times \\ &\quad\times\partial_m\frac{1}{|y_{12}|}\partial_n\partial_p\partial_q\int d^3y_3\,\frac{1}{|y_{23}|}\frac{1}{|y_{31}|}\,, \end{aligned} \tag{4.11}$$

where all the derivatives act on the 2 coordinate.

Using the γ-matrix identity $\gamma^n\gamma^p\gamma^q = \gamma^n\gamma^{pq}+\gamma^n g^{pq}$, we generate a 3D laplacian; acting on $1/|y_{23}|$ this generates another δ-function which allows us trivially to do the integral. We finally obtain

$$\begin{aligned} M\frac{\partial}{\partial M}\left\langle j^{Ak}(y_1)\,j^{Bl}(y_2)\right\rangle &= -\frac{16\,g^2}{3(4\pi^2)(4\pi)^2}\,\mathrm{Tr}\left(T^AT^B\right)\,\mathrm{Tr}\,[P_+\gamma^k\gamma^m\gamma^l\gamma^n]\partial_m\frac{1}{|y_{12}|}\partial_n\frac{1}{|y_{12}|}\,, \\ &= \frac{16\,g^2}{3(4\pi^2)(4\pi)^2}\,\mathrm{Tr}\left(T^AT^B\right)\left(\frac{2}{|y_{12}|^4}J^{kl}(y_{12})\right)\,, \end{aligned} \tag{4.12}$$

where J^{kl} is the conformal jacobian tensor $J^{kl}(y) \equiv \delta^{kl} - 2y^ky^l/y^2$.

We may use the result (4.12) to the calculate the anomalous dimension of j^k. Because j^k is a conformal primary defect vector operator of some scaling dimension Δ, its two-point function must have the form

$$\left\langle j^{Ak}(y_1)\,j^{Bl}(y_2)\right\rangle = -c(g)\,\mathrm{Tr}\,(T^AT^B)\,\frac{M^4}{8\pi^2|My_{12}|^{2\Delta}}\,J^{kl}(y_{12})\,. \tag{4.13}$$

We have normalized the correlator to the free-field value at $g = 0$. To order g^2, we expect that the scaling dimension is $\Delta = 2 + \gamma(g)$ and $c(g) = 1 + g^2a$ for some constant a, where $\gamma(g)$ is the anomalous dimension of j. We may determine γ using the perturbative relation

$$\frac{M^4}{|My_{12}|^{2\Delta}} \sim \frac{1}{|y_{12}|^4}[1 - 2\gamma(g)\log|y_{12}M|]\,. \tag{4.14}$$

We insert this in (4.13), compute the scale derivative $M\partial/\partial M$, and compare with (4.12). We thus identify the anomalous dimension to order g^2 as

$$\gamma(g^2) = \frac{2g^2}{3\pi^2}\,. \tag{4.15}$$

Note that it is positive, as required by unitarity.

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A term of the form (4.13) appearing in $\langle JJ \rangle$ seems to be inconsistent with the conservation of J when $\Delta \neq 2$, so the M-dependence coming from this term has to cancel out in the full $\langle JJ \rangle$ correlator. Let us now show that it is indeed canceled by the contributions from $\langle \mathcal{J} j \rangle$ and $\langle j \mathcal{J} \rangle$. We have

$$\begin{aligned} \left\langle \mathcal{J}^{Ak}(z_1)\, j^{Bl}(y_2) \right\rangle = & -\frac{4\,g^2}{(4\pi^2)^2(4\pi)^3}\,\mathrm{Tr}\,(T^A T^B)\mathrm{Tr}\,[\gamma^\mu\,\gamma^k\,\gamma^\nu\,\gamma^m P_+ \gamma^l P_+\,\gamma^n] \times \\ & \times \int d^3y_3\, d^3y_4\, \frac{1}{|y_{34}|}\left(\partial_\mu \frac{1}{z_{31}^2}\right)\left(\partial_\nu \frac{1}{z_{14}^2}\right)\left(\partial_m \frac{1}{|y_{42}|}\right)\left(\partial_n \frac{1}{|y_{23}|}\right). \end{aligned} \tag{4.16}$$

Again we must regulate the subdivergence, which in this case comes from the divergent $\lambda/\lambda/q$ loop. Note that there is no overlapping divergence, since the limit where the q propagator approaches the $\bar{\Psi}\Psi$ vertex is finite. The subgraph is

$$(P_+\gamma^\mu\gamma^k\gamma^\nu P_+)\,\frac{1}{|y_{34}|}\left(\partial_\mu \frac{1}{z_{31}^2}\right)\left(\partial_\nu \frac{1}{z_{14}^2}\right). \tag{4.17}$$

We are using a convention where the derivative always acts on the first coordinate listed in a propagator. Switching the μ-derivative to the 1 coordinate, we can pull it out to find

$$\begin{aligned} (P_+\gamma^\mu\gamma^k\gamma^\nu P_+)\left[\partial^1_\mu\left(\frac{1}{|y_{34}|}\frac{1}{z_{31}^2}\left(\partial_\nu \frac{1}{z_{14}^2}\right)\right) + \frac{1}{|y_{34}|}\frac{1}{z_{31}^2}\left(\partial_\mu\partial_\nu \frac{1}{z_{14}^2}\right)\right] \equiv (P_+\gamma^\mu\gamma^k\gamma^\nu P_+) \times \\ \times \left[\partial^1_\mu B_\nu + C_{\mu\nu}\right]. \end{aligned} \tag{4.18}$$

For the first term, the vector B_ν is log divergent by power counting, but since it is a vector, a further derivative can be extracted and what remains is finite. Hence the divergence lies in the second piece $C_{\mu\nu}$, on which we now concentrate.

In differential regularization, tensors like $\partial_\mu\partial_\nu$ are split into their trace and traceless parts; the divergence lies entirely in the trace. In our case, the situation is slightly more subtle due to the defect. Since y_3 and y_4 are only three-dimensional points, the traceless tensor that is appropriate is the three-dimensional one $\partial_k\partial_l - \frac{1}{3}\delta_{kl}\nabla^2$. Thus, we are forced to split the tensor $C_{\mu\nu}$ into defect indices and transverse indices separately.

If one of μ and ν is a defect coordinate while the other is not, the projection matrices P_+ annihilate one another. Thus we get

$$(P_+\gamma^\mu\gamma^k\gamma^\nu P_+)\,C_{\mu\nu} = \frac{1}{|y_{34}|}\left(\frac{1}{z_{13}^2}\right)\left[(P_+\gamma^m\gamma^k\gamma^n)\partial^1_m\partial^1_n + (P_+\gamma^x\gamma^k\gamma^x)(\partial_x)^2\right]\left(\frac{1}{z_{14}^2}\right). \tag{4.19}$$

We split $\partial_m\partial_n = \frac{1}{3}\delta_{mn}\nabla^2+$ traceless, and ignore the traceless part. Using $\gamma^x\gamma^k\gamma^x = -\gamma^k$, $\gamma^m\gamma^k\gamma^n\delta_{mn} = -\gamma^k$, and $(\partial_3)^2 = \Box - \nabla^2$ with $\Box$ the 4D laplacian, we arrive at

$$(\gamma^k P_+)\,\frac{1}{|y_{34}|}\left(\frac{1}{z_{13}^2}\right)\left(\frac{2}{3}\nabla^2 - \Box\right)\left(\frac{1}{z_{14}^2}\right). \tag{4.20}$$

To treat the ∇^2 term, we first switch the 3D laplacian to act on the 4 coordinate and then integrate by parts. Terms with an overall ∇_k are not singular, so the relevant term

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involves $\nabla^2(1/|y_{43}|) = -4\pi\delta(y_{43})$. With this δ-function present, we can combine the two 4D propagators and regulate the resulting expression in the standard 4D fashion

$$\left(\frac{1}{z_{13}^2}\right)\left(\frac{1}{z_{14}^2}\right) \to \left(\frac{1}{z_{13}^4}\right) \to -\frac{1}{4}\Box\,\frac{\log M^2 z_{13}^2}{z_{13}^2}\,. \tag{4.21}$$

Now turn to the $\Box$ term in (4.20). The 4D laplacian acts on the propagator to produce $\delta(z_{14})$, which allows us to regulate

$$\frac{1}{|y_{34}|}\frac{1}{z_{13}^2} \to \frac{1}{|y_{34}|^3} \to -\nabla^2\frac{\log M|y_{34}|}{|y_{34}|}\,. \tag{4.22}$$

The total expression for both terms is hence

$$(\gamma^k P_+)\left[\frac{1}{6}(4\pi)\left(\Box\,\frac{\log M^2 z_{13}^2}{z_{13}^2}\right)\delta(y_{43}) - (4\pi^2)\nabla^2\frac{\log M|y_{34}|}{|y_{34}|}\delta(z_{14})\right]. \tag{4.23}$$

Taking $M\partial/\partial M$ to isolate the M-dependent part of the subdivergence, we have

$$\frac{2}{3}(\gamma^k P_+)\,(4\pi^2)(4\pi)\delta(z_{14})\delta(y_{34})\,. \tag{4.24}$$

Substituting the subdivergence back into the total expression for $\langle\mathcal{J}j\rangle$, and recalling that all the terms we dropped are M-independent, we thus have

$$\begin{aligned} M\frac{\partial}{\partial M}\left\langle\mathcal{J}^{Ak}(z_1)\,j^{Bl}(y_2)\right\rangle &= -\frac{8\,g^2}{3(4\pi^2)(4\pi)^2}\,\mathrm{Tr}\,(T^A T^B)\mathrm{Tr}\,[P_+\gamma^k\,\gamma^m\,\gamma^l\,\gamma^n]\times\\ &\quad\times\int d^3y_3\,d^3y_4\,\delta(z_{13})\delta(y_{34})\left(\partial_m\frac{1}{|y_{42}|}\right)\left(\partial_n\frac{1}{|y_{23}|}\right)\\ &= \frac{8\,g^2\,\delta(x_1)}{3(4\pi^2)(4\pi)^2}\,\mathrm{Tr}\,(T^A T^B)\mathrm{Tr}\,[P_+\gamma^k\,\gamma^m\,\gamma^l\,\gamma^n]\times\\ &\quad\times\left(\partial_m\frac{1}{|y_{12}|}\right)\left(\partial_n\frac{1}{|y_{12}|}\right). \end{aligned} \tag{4.25}$$

The contribution from $\langle j\mathcal{J}\rangle$ is identical up to exchanging $x_1 \leftrightarrow x_2$. Consequently from (4.12) and (4.25) we see

$$\begin{aligned} M\frac{\partial}{\partial M}\Big(\delta(x_2)\left\langle\mathcal{J}^{Ak}(z_1)\,j^{Bl}(y_2)\right\rangle + \delta(x_1)\left\langle j^{Ak}(y_1)\,\mathcal{J}^{Bl}(z_2)\right\rangle +\\ +\delta(x_1)\delta(x_2)\left\langle j^{Ak}(y_1)\,j^{Bl}(y_2)\right\rangle\Big) = 0\,, \end{aligned} \tag{4.26}$$

and $\langle J_V J_V\rangle$ has no divergences, as required. Notice that we had to isolate the M-dependent part of $\langle\mathcal{J}j\rangle$ in order to obtain the $\delta(x_1)$. The complete correlation function $\langle\mathcal{J}(z_1)j(y_2)\rangle$ is not localized on the defect in the x_1 coordinate, only the divergent part is.

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5. Anomalous dimensions from gravity

As discussed in section 2, the reduced conformal group SO(3, 2) preserved by the defect is more permissive than the full group SO(4, 2), but still places strong constraints on correlation functions (in this section we specialize to the case of $d = 4$, though the generalization to arbitrary d should be straightforward). The simplest permitted correlators involving 4D operators are the one-point function $\langle \mathcal{O}_4 \rangle$ and the mixed two-point function $\langle \mathcal{O}_4 \mathcal{O}_3 \rangle$. As with two- and three-point functions in ordinary CFT, the coordinate dependence of these correlators is completely specified by the symmetry. It was shown in [11] that the corresponding AdS_4 interactions $\int_{AdS_4} \phi_5$ and $\int_{AdS_4} \phi_5 \psi_4$ lead to dCFT correlation functions with the correct structure.

In this section we consider the gravity calculation of two-point correlation functions between four-dimensional operators. Just as the one-point and mixed two-point cases in dCFT are somewhat analogous to the two- and three-point functions of ordinary CFT, the case of ambient two-point functions is analogous to the ordinary CFT four-point function. As discussed in section 2 it is the simplest correlator admitting a nontrivial coordinate dependence, and this coordinate dependence encodes the dimensions of operators living on the defect.

There are a number of ways to generate contributions to these correlators in the gravity theory with a probe brane. At the leading order in the interaction with the brane (which in the D3/D5 system is the same as leading order in $g_s M$, a disc diagram in string theory), we encounter three classes of diagrams in perturbative calculations, which are drawn in figure 5. Each diagram is independent of the others so it must individually lead to a contribution with the form (2.4) to $\mathcal{M}_{\Delta_1 \Delta_2}(\vec{y}_1, x_1; \vec{y}_2, x_2) \equiv \langle \mathcal{O}_{\Delta_1}(\vec{y}_1, x_1)\, \mathcal{O}_{\Delta_2}(\vec{y}_2, x_2) \rangle$.

Below we present the general analysis for the various types of diagrams. We will only consider the two-point functions of scalars, interacting with other bulk scalars as well as brane scalars; generically intermediate higher spin fields would also contribute to the two-point function of scalar operators but we do not consider them here. We adapt the method of "without really trying" [26] that was created for evaluating CFT four-point functions.

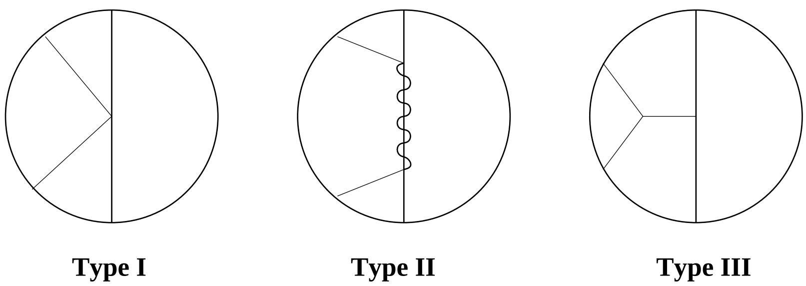

Figure 5: Diagrams appearing in the computation of $\langle \mathcal{O}_4 \mathcal{O}_4 \rangle$. The circle represents the boundary of AdS_5, where we insert the operators, and the vertical straight line crossing it represents the AdS_4 brane. The other solid lines are bulk scalar propagators, and the curly line is a brane scalar propagator.

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We work in euclidean AdS, with quadratic terms normalized as

$$\mathcal{L} = \frac{1}{2\kappa_5^2}\int d^5z\sqrt{-g_5}\left(-\mathcal{R}_5 + \frac{1}{2}\sum_i(\partial_\mu\phi_i\partial^\mu\phi_i + m_i^2\phi_i\phi_i)\right) + \\ + \frac{1}{2\kappa_4^2}\int d^4z\sqrt{-g_4}\left(\frac{1}{2}\sum_j(\partial_{\mu'}\psi_j\partial^{\mu'}\psi_j + m_j^2\psi_j\psi_j)\right). \tag{5.1}$$

The euclidean AdS_5 metric is

$$ds^2 = \frac{1}{z_0^2}\left(dz_0^2 + d\vec{y}^2 + dx^2\right), \tag{5.2}$$

where we have set the AdS curvature radius to one (it is easy to reinstate it if desired), and the AdS_4 metric is the same without the dx^2 term. From here on we will use the letters z and w to denote the AdS coordinates, and d^4z, d^5z will be the volume elements of AdS_4 and AdS_5, respectively.

5.1 Type-I

The type-I diagrams are the simplest, representing contact terms that involve only one integral over an internal point. Moreover, the more complicated diagrams of type-II and type-III can be shown to reduce to type-I diagrams. We shall therefore begin with them.

Assuming that the brane action includes a coupling of bulk fields of the form

$$\lambda_I\int d^4z\sqrt{-g_4}\,\phi_1\phi_2\,, \tag{5.3}$$

we find[8] a contribution to the two-point function $\mathcal{M}_{\Delta_1\Delta_2}$ of the form

$$\mathcal{M}_{\Delta_1\Delta_2} = -\lambda_I\int d^4z\sqrt{-g_4}\,K_{\Delta_1}(z;\vec{y}_1,x_1)K_{\Delta_2}(z;\vec{y}_2,x_2)\,, \tag{5.4}$$

where K_Δ is the standard bulk-to-boundary propagator:

$$K_\Delta(z;\vec{y},x) = C_\Delta\left(\frac{z_0}{x^2+(\vec{y}-\vec{z})^2+z_0^2}\right)^\Delta \equiv C_\Delta\tilde{K}_\Delta(z;\vec{y},x)\,, \tag{5.5}$$

with $C_\Delta = \Gamma(\Delta)/(\pi^2\Gamma(\Delta-2))$. The contribution to the two-point function is thus:

$$\mathcal{M}_{\Delta_1\Delta_2}(\vec{y}_1,x_1;\vec{y}_2,x_2) = -\lambda_I C_{\Delta_1}C_{\Delta_2}I_{\Delta_1\Delta_2}(\vec{y}_1,x_1;\vec{y}_2,x_2)\,, \tag{5.6}$$

where

$$I_{\Delta_1\Delta_2}(\vec{y}_1,x_1;\vec{y}_2,x_2) = \int\frac{dz_0d\vec{z}}{z_0^4}\left(\frac{z_0}{x_1^2+(\vec{y}_1-\vec{z})^2+z_0^2}\right)^{\Delta_1}\left(\frac{z_0}{x_2^2+(\vec{y}_2-\vec{z})^2+z_0^2}\right)^{\Delta_2}. \tag{5.7}$$

[8]Assuming that ϕ_1 and ϕ_2 are different fields; an additional factor of 2 arises if they are the same.

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This will be the standard object, in terms of which we will express the more complicated diagrams. We use translation invariance along the defect to set $\vec{y}_2 = 0$, and denote $\vec{y} \equiv \vec{y}_1$. The integral can be performed using Feynman parameters, giving

$$I_{\Delta_1\Delta_2}(\vec{y}, x_1; 0, x_2) = D_{\Delta_1\Delta_2} \int_0^1 da \frac{a^{\Delta_1 - 1}(1-a)^{\Delta_2 - 1}}{(ax_1^2 + (1-a)x_2^2 + a(1-a)\vec{y}^2)^{(\Delta_1+\Delta_2)/2}}\,, \tag{5.8}$$

where

$$D_{\Delta_1\Delta_2} \equiv \frac{\pi^{3/2}}{2} \frac{\Gamma\left(\frac{\Delta_1+\Delta_2-3}{2}\right)\Gamma\left(\frac{\Delta_1+\Delta_2}{2}\right)}{\Gamma(\Delta_1)\Gamma(\Delta_2)}\,. \tag{5.9}$$

Based on the discussion in section 2 we expect the result of this integral to have the form $I_{\Delta_1\Delta_2} = f(\xi)/x_1^{\Delta_1}x_2^{\Delta_2}$. This implies that if we define $f(\vec{y}, x_1, x_2) = x_1^{\Delta_1}x_2^{\Delta_2}I_{\Delta_1\Delta_2} \times (\vec{y}, x_1; 0, x_2)$, it should be purely a function of ξ, so it should obey

$$\left(x_1 \frac{\partial}{\partial x_1} + x_2 \frac{\partial}{\partial x_2} + 2\vec{y}^2 \frac{\partial}{\partial \vec{y}^2}\right) f(\vec{y}, x_1, x_2) = 0 \tag{5.10}$$

and

$$\left(x_1 \frac{\partial}{\partial x_1} - x_2 \frac{\partial}{\partial x_2} - 2(x_1^2 - x_2^2) \frac{\partial}{\partial \vec{y}^2}\right) f(\vec{y}, x_1, x_2) = 0\,. \tag{5.11}$$

By simple manipulations on the integral (5.8) it is easy to show that these equations are indeed satisfied, so the expressions that we find here are consistent with the form (2.4) of the two-point function, dictated by conformal invariance.

We may then solve (5.8) by simply setting $\vec{y} = 0$ and restoring the full ξ-dependence at the end. We find

$$\begin{aligned} I_{\Delta_1\Delta_2}(\xi, x_1, x_2) &= \frac{\pi^{3/2}\Gamma\left(\frac{\Delta_1+\Delta_2-3}{2}\right)\Gamma\left(\frac{\Delta_1+\Delta_2}{2}\right)}{2x_1^{\Delta_1}x_2^{\Delta_2}\Gamma\left(\Delta_1+\Delta_2\right)} \zeta^{\Delta_1} \cdot {}_2F_1\left(\frac{\Delta_1+\Delta_2}{2}, \Delta_1; \Delta_1+\Delta_2; 1-\zeta^2\right), \\ \zeta &\equiv 1 + 2\xi + 2\sqrt{\xi(\xi+1)}\,, \end{aligned} \tag{5.12}$$

valid for $\xi > 0$. Although this is not manifestly symmetric between Δ_1 and Δ_2, it can be shown to be so using hypergeometric identities. The small-ξ expansion of (5.12) is simply

$$I_{\Delta_1\Delta_2}(\xi \to 0, x_1, x_2) = \frac{\pi^{3/2}\Gamma\left(\frac{\Delta_1+\Delta_2-3}{2}\right)\Gamma\left(\frac{\Delta_1+\Delta_2}{2}\right)}{2x_1^{\Delta_1}x_2^{\Delta_2}\Gamma\left(\Delta_1+\Delta_2\right)} \left(1 + \mathcal{O}(\xi)\right)\,. \tag{5.13}$$

For odd $\Delta_1 + \Delta_2$, (5.12) reduces to nice rational functions of ξ, while for even $\Delta_1 + \Delta_2$ there is a logarithmic term, and the leading large-ξ behavior of $I_{\Delta_1\Delta_2}$ is proportional to $\log(\xi)/x_1^{\Delta_1}x_2^{\Delta_2}\xi^{\max(\Delta_1,\Delta_2)}$. In subsection 5.4 we explain these logarithms as resulting from the anomalous dimensions of the reduced operators appearing in the BOPE of $\mathcal{O}_{\Delta_1}$ and $\mathcal{O}_{\Delta_2}$.

As a simple example, $I_{1\,1}(\xi, x_1, x_2)$ may be evaluated using either (5.8) or (5.12), giving

$$I_{1\,1}(\xi, x_1, x_2) = -\frac{\pi^2}{4x_1x_2\sqrt{\xi(\xi+1)}} \log\left(\frac{\xi + \frac{1}{2} + \sqrt{\xi(\xi+1)}}{\xi + \frac{1}{2} - \sqrt{\xi(\xi+1)}}\right)\,. \tag{5.14}$$

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Note that for $\Delta_1 = \Delta_2 = 1$ the integral (5.7) actually diverges, but we can still assign to it the above value (following from (5.8)) by analytic continuation in Δ_1 and Δ_2. For $\Delta_1 + \Delta_2 = 3$ there is a pole in this analytically continued expression, but it is well-behaved for $\Delta_1 + \Delta_2 > 3$. As another example, we have

$$I_{3\,2}(\vec{y}, x_1; 0, x_2) = \frac{\pi^2}{(2x_1)^3 (2x_2)^2 2(\xi+1)^2} \,. \tag{5.15}$$

Note that $-1 < \xi < 0$ does not occur.

Instead of computing them directly, we can obtain all other $I_{\Delta_1 \Delta_2}$ with integer Δ_1 and Δ_2 by taking derivatives of (5.14) and (5.15). From the expression (5.8) we obtain the relation

$$\frac{\partial}{\partial \vec{y}^2} I_{\Delta_1 \Delta_2} = \frac{1}{4x_1 x_2} \frac{\partial}{\partial \xi} I_{\Delta_1 \Delta_2} = -\left(\frac{2\Delta_1 \Delta_2}{\Delta_1 + \Delta_2 - 3}\right) I_{\Delta_1+1\,\Delta_2+1} \,. \tag{5.16}$$

Furthermore, defining $\tilde{x}^2 \equiv \vec{y}^2 + x_1^2$ and holding it fixed, we can also obtain

$$\left.\frac{\partial}{\partial \vec{y}^2}\right|_{\tilde{x}^2} I_{\Delta_1 \Delta_2} = \frac{2\Delta_1(\Delta_1 + 1)}{\Delta_1 + \Delta_2 - 3} I_{\Delta_1+2,\Delta_2} \,. \tag{5.17}$$

As we shall see, all the other contributions to two-point functions at leading order can be written in terms of the functions $I_{\Delta_1 \Delta_2}$.

It is worth noting that couplings with derivatives can also appear in the action on the brane. For example, consider a term of the form

$$\varsigma_I \int d^4 z \sqrt{-g_4}\, \phi_1 (z_0 \partial_{z_3} \phi_2) \,, \tag{5.18}$$

where the brane is located at $z_3 = 0$ and the restriction of the derivative of ϕ_2 normal to the brane appears in the interaction. This leads to the contribution

$$\mathcal{M}_{\Delta_1 \Delta_2}(\vec{y}_1, x_1; \vec{y}_2, x_2) = -\varsigma_I \int d^4 z \sqrt{-g_4}\, K_{\Delta_1}(z; \vec{y}_1, x_1)\, (z_0 \partial_{z_3} K_{\Delta_2}(z; \vec{y}_2, x_2)) \,, \tag{5.19}$$

where we set $z_3 = 0$ for K_{Δ_2} only after acting with the derivative. We find

$$z_0 \partial_{z_3} \tilde{K}_\Delta(z; \vec{y}, x) = 2x\Delta \tilde{K}_{\Delta+1}(z; \vec{y}, x) \,, \tag{5.20}$$

and correspondingly

$$\mathcal{M}_{\Delta_1 \Delta_2}(\vec{y}_1, x_1; \vec{y}_2, x_2) = -2\varsigma_I \Delta_2 C_{\Delta_1} C_{\Delta_2} [x_2 I_{\Delta_1 \Delta_2+1}(\vec{y}_1, x_1; \vec{y}_2, x_2)] \,. \tag{5.21}$$

From the discussion above it is clear that $x_2 I_{\Delta_1 \Delta_2+1}(\vec{y}_1, x_1; \vec{y}_2, x_2)$ has the correct coordinate dependence to satisfy the conformal invariance condition (2.4).

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5.2 Type-II

We assume a brane interaction between bulk fields ϕ_i, $i = 1, 2$ with associated conformal dimensions Δ_i, and a brane field ψ with associated conformal dimension Δ, of the form

$$\sum_{i=1}^{2} \lambda_{II}^{i} \int d^4z \sqrt{-g_4}\, \phi_i \psi \, . \tag{5.22}$$

The contribution to a two-point function is then

$$\mathcal{M}_{\Delta_1\Delta_2} = \lambda_{II}^{1}\lambda_{II}^{2}(2\kappa_4^2)C_{\Delta_1}C_{\Delta_2} \int \frac{d^4w}{w_0^4} K_{\Delta_2}(w;\vec{y}_2, x_2) B(w;\vec{y}_1, x_1)\,, \tag{5.23}$$

$$B(w;\vec{y}_1, x_1) \equiv \int \frac{d^4z}{z_0^4}\, K_{\Delta_1}(z;\vec{y}_1, x_1)\, G_\Delta(u)\,, \tag{5.24}$$

where $G_\Delta(u)$ is a scalar bulk-to-bulk propagator for the field ψ, depending on the chordal distance between the points z and w,

$$u = \frac{1}{2z_0 w_0}\left((\vec{z} - \vec{w})^2 + (z_0 - w_0)^2\right) \, . \tag{5.25}$$

Note that the points z, w are both on the AdS_4 brane.

We can use a method analogous to the "not really trying" method of [26] to determine $B(w;\vec{y}_1, x_1)$; the idea is to use symmetries and the scalar wave operator to obtain an ordinary differential equation for B. Poincaré invariance requires that B depend only on z_0, x_1 and $|\vec{z} - \vec{y}_1|$. Scaling all coordinates by a constant Λ, u is invariant and hence $B \to \Lambda^{-\Delta_1} B$, implying

$$B(w;\vec{y}_1, x_1) = x_1^{-\Delta_1} J\left(\eta, \frac{z_0}{x_1}\right)\,, \tag{5.26}$$

where

$$\eta \equiv \frac{z_0 x_1}{z_0^2 + x_1^2 + |\vec{z} - \vec{y}_1|^2} \tag{5.27}$$

is a scale-invariant combination of the variables. One can also check that η is invariant under an inversion of all coordinates,

$$z_\mu \to \frac{z_\mu}{z^2}\,, \qquad x_1 \to \frac{x_1}{x_1^2 + y_1^2}\,, \qquad \vec{y}_1 \to \frac{\vec{y}_1}{x_1^2 + y_1^2}\,. \tag{5.28}$$

Under this inversion one finds that B and $x_1^{-\Delta_1}$ transform identically, meaning J is invariant; since z_0/x_1 is not inversion-invariant, we must have simply

$$B(w;\vec{y}_1, x_1) = x_1^{-\Delta_1} J\,(\eta) \, . \tag{5.29}$$

Applying the AdS_4 wave operator obeying $(-\Box + m_\psi^2)\, G_\Delta(u) = \delta(z, w)$ to (5.29), we obtain the ODE for J,

$$\left[(4\eta^4 - \eta^2)\partial_\eta^2 + (8\eta^3 + (d-1)\eta)\partial_\eta + m_\psi^2\right] J(\eta) = \eta^{\Delta_1}\,. \tag{5.30}$$

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Looking for a power series solution of the form

$$J(\eta) = \sum_k b_k \, \eta^k \,, \tag{5.31}$$

we find

$$b_{\Delta_1} = 0 \,, \tag{5.32}$$

$$b_{\Delta_1 - 2} = \frac{1}{4(\Delta_1 - 1)(\Delta_1 - 2)} \,, \tag{5.33}$$

$$b_{k-2} = \frac{(k-\Delta)(k+\Delta-d)}{4(k-1)(k-2)} \, b_k \,, \qquad k = \Delta_1 - 2, \Delta_1 - 4, \ldots, k_{\min} \,, \tag{5.34}$$

with $k_{\min} = \Delta$ and other $b_k = 0$. The total contribution is then

$$\mathcal{M}_{\Delta_1 \Delta_2} = \lambda^1_{II} \lambda^2_{II} (2\kappa_4^2) C_{\Delta_1} C_{\Delta_2} \left\{ \sum_k b_k \, x_1^{k-\Delta_1} I_{k,\Delta_2}(\vec{y}_1, x_1; \vec{y}_2, x_2) \right\} . \tag{5.35}$$

The amplitude has been reduced to a sum of diagrams of type-I multiplied by a function of coordinates. Each term has the correct form (2.4) dictated by defect conformal invariance. This solution requires $\Delta_1 - \Delta$ to be a positive even integer; a similar constraint appeared in [26]. We do not know how to compute other type-II diagrams where this constraint is not satisfied. Of course, we can repeat the same analysis exchanging ϕ_1 and ϕ_2, and if both $\Delta_1 - \Delta$ and $\Delta_2 - \Delta$ are even integers we obtain the same answer in different forms.

5.3 Type-III

Assuming a bulk interaction of the form

$$S_{bulk} = \frac{1}{2\kappa_5^2} \int d^5 z \sqrt{-g_5} \, (\phi_{\Delta_1} \phi_{\Delta_2} \phi_{\Delta_3}) \,, \tag{5.36}$$

and a brane interaction

$$S_{brane} = \lambda \int d^4 z \sqrt{-g_4} \, \phi_{\Delta_3} \,, \tag{5.37}$$

the usual manipulations result in the expression

$$\begin{aligned} \mathcal{M}_{\Delta_1 \Delta_2}(\vec{x}_1, y_1; \vec{x}_2, y_2) &= \lambda \int \frac{d^4 w}{w_0^4} \, A(w; \vec{y}_1, x_1; \vec{y}_2, x_2) \,, \\ A(w; \vec{y}_1, x_1; \vec{y}_2, x_2) &\equiv \int \frac{d^5 z}{z_0^5} \, K_{\Delta_1}(z; \vec{y}_1, x_1) K_{\Delta_2}(z; \vec{y}_2, x_2) \, G_{\Delta_3}(u) \,, \end{aligned} \tag{5.38}$$

where G_Δ is now an AdS_5 bulk-to-bulk propagator, and the chordal distance u now reflects the fact that z is not pinned to the brane,

$$u = \frac{1}{2 z_0 w_0} \left((\vec{z} - \vec{w})^2 + (z_3)^2 + (z_0 - w_0)^2 \right) . \tag{5.39}$$

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Note that the propagator generates a factor $2\kappa_5^2$ that cancels the factor from the coupling. We notice immediately that the function $A(w;1;2)$ is identical to the function A defined in (2.11) of [26]. It involves integrating a point over the complete bulk, and is not aware of the presence of the defect, other than the fact that $w_3 = 0$. Hence one can take the results from that analysis wholesale. We proceed by shifting $\vec{y}_1 \to 0$ and inverting the coordinates into primed coordinates, and we arrive at [26]

$$A(w;1;2) = \frac{1}{(\vec{x}_{12} + y_{12})^{2\Delta_2}} I(\vec{w}' - \vec{y}'_{12}, w'_3 - x'_{12}, w'_0)\,, \tag{5.40}$$

$$I(\vec{y}, x, w_0) = w_0^{\Delta_{12}} \sum_k a_k \left(\frac{w_0^2}{w_0^2 + \vec{y}^2 + x^2} \right)^k, \tag{5.41}$$

where the a_k are given recursively by

$$a_k = 0 \quad \text{for} \quad k \geq \Delta_2\,, \tag{5.42}$$

$$a_{\Delta_2 - 1} = \frac{1}{4(\Delta_1 - 1)(\Delta_2 - 1)}\,, \tag{5.43}$$

$$a_{k-1} = \frac{(k - \frac{\Delta_3}{2} + \frac{\Delta_{12}}{2})(k - 2 + \frac{\Delta_3}{2} + \frac{\Delta_{12}}{2})}{(k-1)(k-1+\Delta_{12})} a_k\,. \tag{5.44}$$

This is the basic result of "not really trying". The series terminates below at $k = (\Delta_3 - \Delta_{12})/2$ if $\Delta_1 + \Delta_2 - \Delta_3$ is a positive even integer. Again, we do not know how to evaluate the diagram if this constraint is not satisfied. However, it seems that the constraint is always satisfied in type-IIB supergravity on $AdS_5 \times S^5$ (though it is not satisfied in the full string theory, nor in other backgrounds).

Assembling the total result for the diagram, including undoing the inversion and translation, one arrives at

$$\mathcal{M}_{\Delta_1\Delta_2}(\vec{y}_1, x_1; \vec{y}_2, x_2) = \lambda C_{\Delta_1} C_{\Delta_2} \sum_k a_k (\vec{y}_{12}^2 + x_{12}^2)^{k-\Delta_2} I_{\Delta_1 - \Delta_2 + k, k}(\vec{y}_1, x_1; \vec{y}_2, x_2)\,. \tag{5.45}$$

As with type-II, the problem has been reduced to a sum of diagrams of type-I, each multiplied by an appropriate function of coordinates. The contribution to (5.45) at a given value of k will have the coordinate dependence

$$(\vec{y}_{12}^2 + x_{12}^2)^{k-\Delta_2} \frac{(2x_2)^{(\Delta_1 - \Delta_2 + k) - k}}{(\vec{y}_{12}^2 + x_{12}^2)^{\Delta_1 - \Delta_2 + k}} F(\xi) = \frac{(2x_2)^{\Delta_1 - \Delta_2}}{(\vec{y}_{12}^2 + x_{12}^2)^{\Delta_1}} F(\xi) = \frac{F(\xi)}{(2x_1\xi)^{\Delta_1}(2x_2)^{\Delta_2}}\,, \tag{5.46}$$

which is indeed the proper form (2.4) for a dCFT two-point function.

Many diagrams will also contain derivative couplings. For example, instead of (5.36) consider the interaction

$$S_{bulk} = \frac{1}{2\kappa_5^2} \int d^5 z \sqrt{-g_5}\, (\partial_\mu \phi_{\Delta_1} \partial^\mu \phi_{\Delta_2} \phi_{\Delta_3})\,, \tag{5.47}$$

along with the brane interaction (5.37). One finds

$$\begin{aligned} \mathcal{M}_{\Delta_1\Delta_2}(\vec{y}_1, x_1; \vec{y}_2, x_2) &= \lambda \int \frac{d^4 w}{w_0^4} \tilde{A}(w; \vec{y}_1, x_1; \vec{y}_2, x_2)\,, \\ \tilde{A}(w; \vec{y}_1, x_1; \vec{y}_2, x_2) &\equiv \int \frac{d^5 z}{z_0^5} \partial_\mu K_{\Delta_1}(z; \vec{y}_1, x_1) \partial^\mu K_{\Delta_2}(z; \vec{y}_2, x_2)\, G_{\Delta_3}(u)\,, \end{aligned} \tag{5.48}$$

where both derivatives are with respect to the z-coordinate. We can process this by means of the identity (A.5) of [33]:

$$\partial_\mu \tilde{K}_{\Delta_1}(z;1)\partial^\mu \tilde{K}_{\Delta_2}(z;2) = \Delta_1\Delta_2 \Big[\tilde{K}_{\Delta_1}(z;1)\tilde{K}_{\Delta_2}(z;2) - 2(\vec{x}_{12}^2 + y_{12}^2) \times \\ \times \tilde{K}_{\Delta_1+1}(z;1)\tilde{K}_{\Delta_2+1}(z;2)\Big]\,. \tag{5.49}$$

This reduces the problem to two diagrams of the non-derivative type. The former piece manifestly produces a result of the correct form (2.4), and it is easy to see that the latter piece does as well.

5.4 From correlators to anomalous dimensions

We have now seen that a wide class of two-point functions in a dCFT can be expressed, in the limit of large 't Hooft coupling and to leading order in the defect interactions, in terms of the functions $I_{\Delta_1\Delta_2}(\xi, x_1, x_2)$. In subsection 5.1, we saw that logarithms of ξ occur in the large ξ limit of $I_{\Delta_1\Delta_2}$ with $\Delta_1 + \Delta_2$ even. Like in the case of four-point functions in the usual AdS/CFT computations [33], these logarithms have an interpretation in terms of corrections to scaling dimensions of "intermediate states".

As discussed in subsection 2.2, a primary defect operator $\mathcal{O}_n(\vec{y})$ of dimension Δ_n appearing in the BOPE of both $\mathcal{O}_{\Delta_1}$ and $\mathcal{O}_{\Delta_2}$ will show up in the large-ξ expansion of $\langle \mathcal{O}_{\Delta_1}\, \mathcal{O}_{\Delta_2}\rangle$ as a term of order $1/\xi^{\Delta_n}$. Thus, if the dimension Δ_n is independent of $g_s M$ we should find a power law behavior, while if $\Delta_n = \Delta_n^{(0)} + \Delta_n^{(1)} g_s M + \ldots$ the expansion of the correlation function will behave as

$$\frac{1}{\xi^{\Delta_n}} = \frac{1}{\xi^{\Delta_n^{(0)}}}(1 - \Delta_n^{(1)} g_s M \log \xi + \ldots)\,. \tag{5.50}$$

In this way, subleading contributions to two-point functions can exhibit logarithmic corrections at first order in $g_s M$. This is completely analogous to the way that logarithms appear in four-point functions in the standard computation of AdS/CFT correlation functions, where they are related to anomalous dimensions of intermediate states appearing in the OPE.

This argument suggests that such a subleading correction can only appear if the leading contribution (at zeroth order in $g_s M$) is non-zero, but actually this is not necessarily the case. If more than one primary operator $\mathcal{O}_n$ of equal dimension $\Delta^{(0)}$ appears in the BOPEs of $\mathcal{O}_{\Delta_1}$ and $\mathcal{O}_{\Delta_2}$, it is possible for the leading contributions of the operators to cancel out precisely, and then the second term in (5.50) can actually be the leading term in the two-point function (if the different primary operators have different $\Delta^{(1)}$'s).

In the case of four-point functions, anomalous dimensions arose for intermediate composite operators of the form $[\mathcal{O}_1(x)\mathcal{O}_2(x)]$, even when $\mathcal{O}_1$ and $\mathcal{O}_2$ themselves did not have anomalous dimensions. This can occur because of the need to regularize the operator product when the two operators approach each other, for instance by a point-splitting regularization. The regularized composite operator formed from two chiral operators need

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not be chiral, and hence need not be protected from acquiring anomalous dimension. In the dCFT case we similarly find anomalous dimensions for “reduced operators” even when the ambient operators have no anomalous dimensions.

For instance, consider an ambient operator $\mathcal{O}(\vec{y},x)$ of integer dimension Δ. The two-point function of this operator at zeroth order in g_sM is just the two-point function without the defect. This has a simple BOPE interpretation in which the intermediate states are just the coefficients in the Taylor expansion of $\mathcal{O}$ around the defect, of dimensions $\Delta+n$ for $n=0,1,2,\ldots$. Now, suppose that at first order in g_sM we have a type-I diagram for the dual field. This diagram gives a contribution proportional to $I_{\Delta\Delta}$ to the two-point function, and the leading large-ξ contribution of this diagram to the correlation function behaves as $\lambda \log\xi/\xi^{\Delta}x_1^{\Delta}x_2^{\Delta}$ with $\lambda \propto g_sM$. Using (5.50) we interpret such a contribution as resulting from the fact that the reduced operator $[\mathcal{O}](\vec{y}) \equiv \lim_{x\to 0}\mathcal{O}(\vec{y},x)$, which had dimension Δ at leading order in g_sM, now acquires an anomalous dimension proportional to λ. Again, it is natural to interpret the fact that the reduced operator $[\mathcal{O}](\vec{y})$ acquires an anomalous dimension even when the ambient operator $\mathcal{O}(\vec{y},x)$ does not as related to the fact that the limit $\lim_{x\to 0}\mathcal{O}(\vec{y},x)$ is non-trivial once interactions with the defect are introduced, and a regularization method such as point-splitting is required to define the reduced operator, which can lead to an anomalous dimension for these operators.

Similarly, the higher powers of $1/\xi$ in the expansion of two-point functions can be related to anomalous dimensions of the higher reduced operators $[\partial_x^n\mathcal{O}](\vec{y}) \equiv \lim_{x\to 0}\partial_x^n\mathcal{O}\times(\vec{y},x)$. In general, operators involving the defect fields will also appear in the BOPE expansion of the two-point function, but they cannot appear in the two-point function at first order in g_sM since their BOPE coefficients are themselves of order g_sM. Thus, at leading order in g_sM the logarithmic terms in the two-point functions just teach us about the anomalous dimensions of the reduced operators. As discussed above, these are relevant for identifying locally localized fields.

We can also discuss the small-ξ limit of the correlation functions we find. As described in section 2, this is related to one-point functions of intermediate operators appearing in the ambient OPE of $\mathcal{O}_{\Delta_1}$ and $\mathcal{O}_{\Delta_2}$. This interpretation is particularly clear for the case of type-III diagrams. These diagrams involve a bulk coupling of some field ϕ_{Δ_3} to $\phi_{\Delta_1}\phi_{\Delta_2}$, which is related to the OPE coefficient of the corresponding operator $\mathcal{O}_{\Delta_3}$ in the OPE of $\mathcal{O}_{\Delta_1}$ with $\mathcal{O}_{\Delta_2}$, and they involve a one-point function of ϕ_{Δ_3} on the brane, which is directly related to the one-point function of $\mathcal{O}_{\Delta_3}$. It is easy to check that the coefficients involved in these relations all match, so the small-ξ limit of these diagrams agrees with our expectations. The interpretation of the small-ξ behavior of type-I and II diagrams is less straightforward; the leading operator visible in the OPE expansion of diagrams of these types has dimension 2Δ, and can be identified with $[\mathcal{O}_\Delta\mathcal{O}_\Delta]$.

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A. Field theory conventions

Here we present the propagators necessary for evaluating the correlation functions in section 4. The toy dQFT consists of the $d = 4$ fermions λ_{im} and the defect fields Ψ^i, q^m, $(i, m = 1, 2)$. We are not careful about whether the indices m, i are raised or lowered, but move them around for notational convenience. We use the spinor conventions of [11].

The Ψ^i and q^m are complex. The λ_{im} are composed of Majorana fields λ, χ^A, $A = 1, 2, 3$:

$$\lambda_{im} \equiv \lambda\delta_{im} - i\chi^A\sigma^A_{im}\,, \qquad \bar{\lambda}_{mi} \equiv \bar{\lambda}\delta_{mi} + i\bar{\chi}^A\sigma^A_{mi}\,, \tag{A.1}$$

and the kinetic terms for the λ, χ^A are canonical:

$$-\frac{i}{4}\bar{\lambda}^{mi}\gamma^\mu\partial_\mu\lambda^{im} = -\frac{i}{2}\bar{\lambda}\gamma^\mu\partial_\mu\lambda - \frac{i}{2}\bar{\chi}^A\gamma^\mu\partial_\mu\chi^A\,. \tag{A.2}$$

The action (4.1) has three continuous symmetries, the defect $\mathrm{U}(1)_B$, under which q and Ψ both have charge 1, and $\mathrm{SU}(2)_V \times \mathrm{SU}(2)_H$ (acting on the indices i and m, respectively), under which $\Psi \to g_V\Psi$, $q \to g_H q$, and $\lambda \to g_V\lambda g_H^\dagger$, with $T^A = \sigma^A/2$ the generators of $\mathrm{SU}(2)$, leading to the currents (4.1).

We now consider the propagators, rotated to euclidean space. For λ_{im}, one processes the $\mathrm{SU}(2)_V \times \mathrm{SU}(2)_H$ indices according to

$$\begin{aligned}
\langle\lambda_{im}(z_1)\,\bar{\lambda}_{nj}(z_2)\rangle &= 2\langle\lambda(z_1)\,\bar{\lambda}(z_2)\rangle\delta_{ij}\delta_{mn}\,,\\
\langle\lambda_{im}(z_1)\,\lambda_{jn}(z_2)\rangle &= 2\langle\lambda(z_1)\,\lambda(z_2)\rangle\epsilon_{ij}\epsilon_{mn}\,,\\
\langle\bar{\lambda}_{mi}(z_1)\,\bar{\lambda}_{nj}(z_2)\rangle &= 2\langle\bar{\lambda}(z_1)\,\bar{\lambda}(z_2)\rangle\epsilon_{ij}\epsilon_{mn}\,,
\end{aligned} \tag{A.3}$$

where we suppressed spinor indices; the propagators on the right-hand side those of an ordinary Majorana particle, evaluated below. In evaluating the above we used the Pauli matrix identity:

$$\sigma^A_{ij}\sigma^A_{kl} = \delta_{ij}\delta_{kl} - 2\epsilon_{ik}\epsilon_{jl} = 2\delta_{il}\delta_{jk} - \delta_{ij}\delta_{kl}\,. \tag{A.4}$$

The massless 4D and 3D scalar propagators are:[9]

$$\langle X(z_1)\,\bar{X}(z_2)\rangle \equiv \Delta_{12} = \frac{1}{4\pi^2}\frac{1}{(z_{12})^2}\,, \qquad \langle q(y_1)\,\bar{q}(y_2)\rangle \equiv \delta_{12} = \frac{1}{4\pi}\frac{1}{|y_{12}|}\,. \tag{A.5}$$

[9] Of course there is no field X in this model, but it is nonetheless useful to define the 4D scalar propagator.

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The massless 4D Fermi propagator is:

$$\langle\lambda(z_1)\,\bar{\lambda}(z_2)\rangle \equiv S_{12} = \gamma^\mu\partial_\mu\Delta_{12}\,. \tag{A.6}$$

Since λ is Majorana, we also have nonvanishing $\langle\lambda_{\alpha a}(z_1)\,\lambda_{\beta b}(z_2)\rangle$ and $\langle\bar{\lambda}_{\alpha a}(z_1)\,\bar{\lambda}_{\beta b}(z_2)\rangle$, determined by $\bar{\lambda} = \lambda^T\gamma^0$. The massless 3D Fermi propagator is:

$$\langle\Psi(y_1)\,\bar{\Psi}(y_2)\rangle \equiv s_{12} = \rho^k\partial_k\,\delta_{12}\,, \tag{A.7}$$

which can be written as in a 4D notation as

$$\langle\Psi^i(z_1)\bar{\Psi}^j(z_2)\rangle = \delta^{ij}(\gamma^k P_+)\,\partial_k\delta_{12} \equiv \delta^{ij}\,\hat{s}_{12}\,, \tag{A.8}$$

where we define the projection matrices $P_\pm$

$$P_\pm \equiv \frac{1}{2}\left(1 \pm \gamma^5\gamma^x\right), \tag{A.9}$$

where $\gamma^5 \equiv -i\gamma^0\gamma^1\gamma^2\gamma^x$ is the chirality matrix. The projection matrices (A.9) obey

$$[P_\pm, \gamma^k] = 0\,, \qquad \gamma^x P_\pm = P_\mp\gamma^x\,, \qquad P_+^2 = P_-^2 = 1\,, \qquad P_+P_- = 0\,. \tag{A.10}$$

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